Helsinki University of Technology, Digital Systems Laboratory TJ

Tik-79.144 Logic in computer science: foundations

Examination, May 22, 1996

Assignment 1 Answer and justify briefly.

(a) Derive a disjuncitve normal form for the sentence $\neg(\neg A \rightarrow (B \wedge C))$.

(b) Does the following hold: the set

$$\{P(f(y, g(z)), h(b)), P(f(h(x), g(a)), v), P(f(h(b), g(z)), y)\}$$

is unifiable (x, y ja z are variables).

(c) Does the following hold: if $\{\phi\} \models \psi$ and $\models \neg\psi$, then $\models \neg\phi$.

(d) Does the following hold: semantic tableaux are a sound and complete proof method for propositional logic.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models (A \rightarrow (B \vee C)) \rightarrow (\neg B \rightarrow (\neg C \rightarrow \neg A))$

(b) $\{\exists x P(x), \exists x \forall y (P(x) \rightarrow Q(y))\} \models \forall y Q(y)$

(c) $\{\forall x(P(x) \rightarrow R(x)), \forall x(P(x) \rightarrow \neg R(x))\} \models \forall x \neg P(x)$

Tableau proofs must contain all intermediary steps !!!

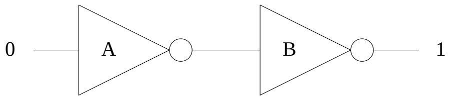
Assignment 3 Show that the sentence $\forall x(\exists y R(x, y) \vee \forall z \neg R(x, z))$ is valid using linear resolution.

Assignment 4 An inverter is a digital circuit which outputs 1 whenever the input is 0 and which outputs 0 whenever the input is 1.

(a) Define the normal operation of an inverter using sentences of predicate logic. Only the predicates $I(x)$: "x is an inverter", $J(x, y)$: "the input of x is y" and $O(x, y)$: "the output of x is y" are allowed.

Hint: you should provide (i) sentences that define how the input of an inverter determines the output of the inverter and (ii) sentences that define uniqueness of inputs and outputs (for any digital circuit).

(b) *Extend* the above set of sentences by sentences describing the combination of inveterters A and B below with the specified input and output.



Show that the *extended* set of sentences is contradictory using semantic tableaux. What can be concluded upon this inconsistency?

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.