Helsinki University of Technology, Digital Systems Laboratory TJ

Tik-79.144 Logic in computer science: foundations

Examination, January 18, 1996

Assignment 1 Answer and justify briefly.

(a) Derive a conjunctive normal form for the sentence $(A \to B) \to C$.

(b) Does the following hold: the set $\{P(x, f(g(a, z)), y), P(f(y), x, z)\}$ is unifiable (x, y ja z are variables).

(c) Does the following hold: if $\{\phi_1, \ldots, \phi_n\} \not\models \phi$, then $\not\models \phi_1 \wedge \ldots \wedge \phi_n \to \phi$[1].

(d) Does the following hold: predicate logic is decidable.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models (A \to (B \vee C)) \to (\neg B \to (\neg C \to \neg A))$

(b) $\{\exists x P(x), \forall x \forall y (P(x) \to P(y))\} \models \forall x P(x)$

(c) $\{\forall x (P(x) \to R(x)), \forall x (Q(x) \to \neg R(x))\} \models \forall x (P(x) \vee Q(x))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 Let R be a binary predicate symbol whose interpretation is a relation $R^{\mathcal{A}} \subseteq A \times A$ (the set A is the domain of the structure $\mathcal{A}$). Below there is a table of sentences defining different properties for the relation $R^{\mathcal{A}}$.

Property	Definition
reflexivity	$\forall x R(x, x)$
irreflexivity	$\forall x \neg R(x, x)$
symmetry	$\forall x \forall y (R(x, y) \to R(y, x))$
asymmetry	$\forall x \forall y (R(x, y) \to \neg R(y, x))$
transitivity	$\forall x \forall y \forall z (R(x, y) \wedge R(y, z) \to R(x, z))$
connectivity	$\forall x \forall y (R(x, y) \vee R(y, x))$

(a) Let $A = \{1, 2, 3\}$. Give $R^{\mathcal{A}}$ such that the sentences defining reflexivity, symmetry and connectivity are satisfied.

(b) Give a tableaux proof on basis of which an irreflexive and transitive relation is also asymmetric.

Assignment 4 Define the predicates $P(x)$ "x is even" and $Q(x, y)$ "x is greated than y" for natural numbers. Represent natural numbers in terms of the constant 0 and the successor function s. Use resolution to find a natural number that is even and greated than 1.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.

[1] In the original examination this sentence was misprint as $\phi_1 \wedge \ldots \wedge \phi_n \to \psi$.