Tik-79.144 Logic in computer science: foundations

Examination on the 14th of December in 1995

Assignment 1 Answer and justify briefly.

(a) Does the following hold: if $\models \phi$, then $\models \psi \rightarrow \phi$ for all sentences ψ.

(b) Give a structure in which the sentence $\exists x \exists y (P(x) \rightarrow Q(y))$ is false.

(c) Does the following hold: if $\Sigma \not\models \phi$ and $\Gamma \subseteq \Sigma$, then $\Gamma \not\models \phi$
(Σ and Γ are sets of sentences).

(d) Does the following hold: the sentece $(\exists x P(x)) \rightarrow (\forall x P(x))$ is a theorem.

Assignment 2 Give a tableaux proof, or give a valuation/structure showing the claim does not hold.

(a) $\models ((A \rightarrow (\neg B \vee C)) \wedge \neg C) \rightarrow (B \rightarrow \neg A)$

(b) $\{\forall x(P(x) \rightarrow R(x)), \forall x(Q(x) \rightarrow \neg R(x))\} \models \forall x(P(x) \rightarrow Q(x))$

(c) $\models (\forall x(P(x) \rightarrow \neg Q(x))) \rightarrow ((\exists x\, Q(x)) \rightarrow (\exists x\, \neg P(x)))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 Show that the sentence $(\exists x \forall y P(x, y)) \rightarrow (\forall y \exists x P(x, y))$ is valid using linear resolution.

Assignment 4 The quantifier $\exists! x$ means that "there is only one x". The claim $\exists! x\, \phi(x)$ can be expressed in predicate logic as the sentence

$$(\exists x \phi(x)) \wedge (\forall x \forall y (\phi(x) \wedge \phi(y) \rightarrow x = y)).$$

Formalize the following sentences in predicate logic:

1. There is only one immortal.
2. All gods are immortal.
3. All immortals are gods.

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4. There is only one god.

Give a tableaux proof on basis of which the sentence 4 is a logical consequence of the sentences 1-3.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.