Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 Answer and justify exactly (at most half a page per item).

(a) True or false: Sheffer’s stroke \( \mid \) is definable in terms of Peirce’s arrow \( \downarrow \).

(b) True or false: if \( \models \phi \lor \psi \), then \( \models \phi \) or \( \models \psi \).

(c) True or false: the empty clause \( \square \) can be obtained from the clauses \( \{ A, \neg B \} \) and \( \{ \neg A, B \} \) by resolution.

(d) True or false: a proof method \( M \) is complete, if every sentence provable by \( M \) is valid.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) \( \models (A \rightarrow (B \lor C)) \rightarrow (\neg B \rightarrow (\neg C \rightarrow \neg A)) \)

(b) \( \models \forall x \exists y (R(x,y) \rightarrow (\forall y (\neg S(y) \rightarrow \exists x R(x,y)) \rightarrow \exists x S(x)) \)

(c) \( \{ \forall x \exists y (P(x) \rightarrow Q(y)), \forall x P(x) \} \models \forall y Q(y) \)

Tableau proofs must contain all intermediary steps !!!

Assignment 3

(a) Derive a clausal form for the sentence

\( \neg(\forall x P(x) \rightarrow \forall x \exists y Q(x,y)) \lor \forall y P(y). \)

Try to make it as simple as possible.

(b) Consider the following program \( P \):

\[ v = 0; z = 0; \text{while}(! (z == y)) \{ z = z + 1; v = v - 1 \}; v = v + x \]

Use weakest preconditions and a suitable invariant to establish

\( \models_P [\text{true}] P [v == x - y]. \)

Assignment 4 Formalize the following claims in terms of predicate logic:

1. If a brick is on another brick, it is not on the table.
2. Every brick is on the table or on another brick.
3. No brick is on a brick which is also on some other brick.
4. If a brick is on another brick, then the latter brick is on the table.

Use resolution to show that the fourth sentence is a logical consequence of the first three sentences.