Helsinki University of Technology, Laboratory for Theoretical Computer Science TJ

T-79.144 Logic in Computer Science: Foundations

Examination, October 25, 2005

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 Answer and justify exactly (at most half a page per item).

(a) True or false: if $\models \phi \vee \psi$, then $\models \phi$ or $\models \psi$.

(b) True or false: a conjunctive normal form ϕ of a sentence in predicate logic is logically equivalent to the form ϕ' obtained from ϕ by Skolemization.

(c) True or false: in propositional logic, at most 16 semantically different binary connectives can be defined.

(d) True or false: predicate logic is decidable.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models (A \wedge B) \vee (\neg A \wedge C) \rightarrow (A \rightarrow B) \wedge (\neg A \rightarrow C)$

(b) $\models \forall x(P(x) \leftrightarrow \neg Q(x)) \leftrightarrow \neg \exists x(P(x) \leftrightarrow Q(x))$

(c) $\{\forall x(P(x) \rightarrow R(x)), \forall x(\neg Q(x) \rightarrow \neg R(x))\} \models \forall x(P(x) \rightarrow Q(x))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3

(a) Derive a clausal form for the sentence

$$\neg(\forall x P(x) \rightarrow \forall x \exists y Q(x,y)) \vee \neg \forall y P(y).$$

Try to make it as simple as possible.

(b) Consider the following program `P`:

```
v=0; z=0; while(!(z==y)) {z=z+1; v=v-1}; v=v+x
```

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\texttt{true}]\ \texttt{P}\ [\texttt{v==x-y}].$$

Assignment 4 Let the predicate $P(x)$ mean that a person x shaves himself, and let the term $f(x)$ refer to the person who is the father of a person x.

(a) Use predicate logic to express the following claim: if a person shaves himself, but his grandfather does not shave himself, then some person shaves himself, but his father does not shave himself.

(b) Use resolution to show that the claim in (a) is valid.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.