Helsinki University of Technology, Laboratory for Theoretical Computer Science TJ

T-79.144 Logic in Computer Science: Foundations

Examination, August 15, 2005

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 Answer and justify exactly (at most half a page per item).

(a) True or false: the empty clause $\Box$ can be obtained from the clauses $\{A, \neg B\}$ and $\{\neg A, B\}$ by resolution.

(b) True or false: if $\models \phi$ and $\phi \wedge \neg\psi$ is unsatisfiable, then $\models \psi$.

(c) True or false: in propositional logic, at most 16 semantically different binary connectives can be defined.

(d) True or false: an existential quantifier can be moved outside an implication as follows: $(\exists x \phi(x) \rightarrow \psi)$ is rewritten as $\forall y(\phi(y) \rightarrow \psi)$ where y is a variable not appearing in formulas $\phi(x)$ and ψ.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models (A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow C) \rightarrow (A \rightarrow B))$

(b) $\models (\forall x(P(x) \rightarrow \neg Q(x))) \rightarrow ((\exists x Q(x)) \rightarrow (\exists x \neg P(x)))$

(c) $\{\forall x(P(x) \rightarrow R(x)), \neg\exists x(\neg R(x) \wedge Q(x))\} \models \forall x(P(x) \vee Q(x) \rightarrow R(x))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3

(a) Derive a clausal form for the sentence

$$\neg(\neg\exists y E(y) \rightarrow \forall y(\exists x E(x) \rightarrow E(y))).$$

Try to make it as simple as possible.

(b) Consider the following program `P`:

```
z=0; v=x; while(!(z==y)) {z=z+1; v=v-1}
```

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\texttt{true}]\ \texttt{P}\ [\texttt{v==x-y}].$$

Assignment 4 Formalize the following claims in terms of predicate logic:

1. If a brick is on another brick, it is not on the table.
2. Every brick is on the table or on another brick.
3. No brick is on a brick which is also on some other brick.

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4. If a brick is on another brick, then the latter brick is on the table.

Use resolution to show that the fourth sentence is a logical consequence of the first three sentences.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.