Assignment 1 Answer and justify exactly (at most half a page per item).

(a) True or false: a proof method M is sound, if every valid sentence \( \varphi \) is provable using the method M.

(b) True or false: a conjunctive normal form \( \varphi \) of a sentence in predicate logic is logically equivalent to the form \( \varphi' \) obtained from \( \varphi \) by Skolemization.

(c) True or false: a sentence \( \varphi \) has at most as many subsentences as it has atomic sentences and connectives (\( \neg, \land, \lor, \to, \leftrightarrow \)).

(d) True or false: if \( \Sigma \not\models \varphi \), then \( \Sigma \models \neg \varphi \).

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) \( \models (A \lor B \to C) \to \neg((A \land \neg C) \lor \neg(B \to C)) \)

(b) \( \{\forall x \forall y (R(x,y) \to R(y,x))\} \models \forall x R(a,x) \)

(c) \( \{\forall x (P(x) \to Q(x) \lor R(x)), \exists x R(x)\} \models \forall x (\neg Q(x) \to \neg P(x)) \)

Tableau proofs must contain all intermediary steps !!!!

Assignment 3

(a) Derive a clausal form for the sentence

\[ \neg(\forall x P(x) \to \forall x \exists y Q(x,y)) \lor \forall y P(y). \]

Try to make it as simple as possible.

(b) Consider the following program \( P \):

\[ v=0; z=0; \text{while}(! (z==y)) \{ z=z+1; v=v-1 \}; v=v+x \]

Use weakest preconditions and a suitable invariant to establish

\[ \models_p [\text{true}] P [v== x-y]. \]

Assignment 4 Let us represent natural numbers 0, 1, 2, … with ground terms 0, \( s(0) \), \( s(s(0)) \), … built of a constant symbol 0 and a function symbol \( s \) which is interpreted as the function \( s(x) = x + 1 \) for natural numbers \( x \).

(a) Let the predicates \( J_2(x), J_3(x) \) and \( J_6(x) \) mean that a natural number \( x \) is divisible by two, three and six, respectively. Use predicate logic to define these predicates such that the definition of the predicate \( J_6 \) is based on the definitions of the predicates \( J_2 \) and \( J_3 \).

(b) Use resolution to show that if a natural number \( x \) is divisible by two and three, then the natural number \( x+6 \) is divisible by six.