Helsinki University of Technology, Laboratory for Theoretical Computer Science TJ

T-79.144 Logic in Computer Science: Foundations

Examination, February 7, 2005

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 Answer and justify exactly (at most half a page per item).

(a) True or false: it holds for every set of sentences Σ and every sentence ϕ that if $\Sigma \models \neg\phi$, then $\Sigma \cup \{\phi\}$ is unsatisfiable.

(b) True or false: Sheffer's stroke $|$ is definable using Peirce's arrow $\downarrow$.

(c) True or false: a conjunctive normal form ϕ of a sentence in predicate logic is logically equivalent to the form ϕ' obtained from ϕ by Skolemization.

(d) True or false: if a sentence ϕ is provable using a sound proof method M_1, then it is also provable using a complete proof method M_2.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models (A \vee B \rightarrow C) \rightarrow (A \rightarrow C) \wedge (\neg C \rightarrow \neg B)$

(b) $\{\forall x \forall y (R(x,y) \rightarrow R(y,x))\} \models \forall x R(a,x)$

(c) $\{\forall x \neg (A(x) \leftrightarrow B(x)), \forall y A(y) \vee \forall y \neg A(y)\} \models \forall z B(z) \vee \forall z \neg B(z)$

Tableau proofs must contain all intermediary steps !!!

Assignment 3

(a) Derive a clausal form for the sentence

$$\neg(\forall x \forall y \neg B(y,x) \wedge \exists x (C(x) \rightarrow A(x))).$$

Try to make it as simple as possible.

(b) Consider the following program `P`:

```
v=0; v=v-x; z=y; while(!(z==0)) {z=z-1; v=v+1}
```

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\texttt{true}]\ \texttt{P}\ [\texttt{v==y-x}].$$

Assignment 4 Consider a ternary predicate $P(x,y,z)$ meaning that the parents of a person x are y and z. Using this predicate, define the binary predicate $R(x,y)$ which means that x is a relative of y. Give a resolution proof that Kerttu is a relative of Kustaa using the following database in addition to your definition.

$$P(\text{kerttu}, \text{jaakoppi}, \text{hanna})$$
$$P(\text{jaakoppi}, \text{reino}, \text{lahja})$$
$$P(\text{kustaa}, \text{salme}, \text{reino})$$

Hint: relatives have an ancestor in common!

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.