Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1

Answer and justify exactly (at most half a page per item).

(a) True or false: it holds for every set of sentences $\Sigma$ and every sentence $\phi$ that if $\Sigma \models \neg \phi$, then $\Sigma \cup \{\phi\}$ is unsatisfiable.

(b) True or false: Sheffer’s stroke $\mid$ is definable using Peirce’s arrow $\downarrow$.

(c) True or false: a conjunctive normal form $\phi$ of a sentence in predicate logic is logically equivalent to the form $\phi'$ obtained from $\phi$ by Skolemization.

(d) True or false: if a sentence $\phi$ is provable using a sound proof method $M_1$, then it is also provable using a complete proof method $M_2$.

Assignment 2

Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models (A \lor B \rightarrow C) \rightarrow (A \rightarrow C) \land (\neg C \rightarrow \neg B)$

(b) $\{\forall x \forall y (R(x, y) \rightarrow R(y, x))\} \models \forall x R(a, x)$

(c) $\{\forall x \neg (A(x) \leftrightarrow B(x)), \forall y A(y) \lor \forall y \neg A(y)\} \models \forall z B(z) \lor \forall z \neg B(z)$

Tableau proofs must contain all intermediary steps !!!

Assignment 3

(a) Derive a clausal form for the sentence

$$\neg (\forall x \forall y \neg B(y, x) \land \exists x (C(x) \rightarrow A(x))).$$

Try to make it as simple as possible.

(b) Consider the following program $P$:

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v = 0; v = v - x; z = y; while (! (z == 0)) {z = z - 1; v = v + 1}
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Use weakest preconditions and a suitable invariant to establish

$$\models_p [\text{true}] P [v == y - x].$$

Assignment 4

Consider a ternary predicate $P(x, y, z)$ meaning that the parents of a person $x$ are $y$ and $z$. Using this predicate, define the binary predicate $R(x, y)$ which means that $x$ is a relative of $y$. Give a resolution proof that Kerttu is a relative of Kustaa using the following database in addition to your definition.

$$P(\text{kerttu}, \text{jaakoppi}, \text{hanna})$$

$$P(\text{jaakoppi}, \text{reino}, \text{lahja})$$

$$P(\text{kustaa}, \text{salme}, \text{reino})$$

Hint: relatives have an ancestor in common!