Helsinki University of Technology, Laboratory for Theoretical Computer Science TJ T-79.144 Logic in Computer Science: Foundations Examination, December 17, 2004

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 Answer and justify exactly (at most half a page per item).

- (a) True or false: the set $\{P(f(y), g(z, f(f(y)))), P(x, g(x, z))\}$ is unifiable.
- (b) True or false: if Σ₁ and Σ₂ are sets of sentences such that Σ₁ ⊆ Σ₂ and φ is a sentence such that Σ₁ ⊨ φ, then also Σ₂ ⊨ φ.
- (c) True or false: a proof method M is sound, if every valid sentence ϕ is provable using the method M.
- (d) True or false: the satisfiability problem of propositional logic is NP-complete.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

- (a) $\models \neg (A \land \neg B) \land (\neg C \to A) \to (A \land B) \lor (\neg A \land C)$
- (b) $\{\forall x(P(x) \to Q(x)), \forall x(Q(x) \to R(x))\} \models \forall x(\neg P(x) \to \neg R(x))$
- (c) $\{\forall x \neg (A(x) \leftrightarrow B(x)), \forall y A(y) \lor \forall y \neg A(y)\} \models \forall z B(z) \lor \forall z \neg B(z)$

Tableau proofs must contain all intermediary steps !!!

Assignment 3

(a) Derive a clausal form for the sentence

$$\neg(\forall x P(x) \to \forall x \exists y Q(x,y)) \lor \neg \forall y P(y).$$

Try to make it as simple as possible.

(b) Consider the following program P:

 $v=0; z=0; while(!(z==y)) \{z=z+1; v=v-1\}; v=v+x$

Use weakest preconditions and a suitable invariant to establish

 $\models_p [true] P [v == x - y].$

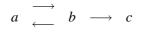
Assignment 4 A *directed* graph consists of a set of nodes connected by *directed* arcs. Let us assume that the nodes of the graph are named with constants $\{a, b, ...\}$ while the arcs of the graph are represented using a predicate A(x, y) = "there is an arc from the node *x* to the node *y*".

(a) Define the predicates

C(x,y) = "there is a connection from the node x to the node y" and L(x) = "the graph has a loop that goes through the node x"

by taking the direction of arcs into account.

(b) Describe the directed graph given below using the predicate *A*. Use resolution to show that the sentence $\exists x(L(x) \land C(x,c))$ is a logical consequence of your description and the definitions of predicates *C* and *L*.



The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.

Feedback: http://www.tcs.hut.fi/Studies/T-79.144/