Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 Answer and justify exactly (at most half a page per item).
(a) True or false: the set \{P(f(y), g(z, f(f(y)))), P(x, g(x, z))\} is unifiable.
(b) True or false: if \(\Sigma_1\) and \(\Sigma_2\) are sets of sentences such that \(\Sigma_1 \subseteq \Sigma_2\) and \(\phi\) is a sentence such that \(\Sigma_1 \models \phi\), then also \(\Sigma_2 \models \phi\).
(c) True or false: a proof method \(M\) is sound, if every valid sentence \(\phi\) is provable using the method \(M\).
(d) True or false: the satisfiability problem of propositional logic is \(NP\)-complete.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).
(a) \(\models \neg(A \land \neg B) \land (\neg C \rightarrow A) \rightarrow (A \land B) \lor (\neg A \land C)\)
(b) \(\{\forall x(P(x) \rightarrow Q(x)), \forall x(Q(x) \rightarrow R(x))\} \models \forall x(\neg P(x) \rightarrow \neg R(x))\)
(c) \(\{\forall x(\neg(A(x) \leftrightarrow B(x))), \forall y A(y) \lor \forall y \neg A(y)\} \models \forall z B(z) \lor \forall z \neg B(z)\)

Tableau proofs must contain all intermediary steps !!!

Assignment 3
(a) Derive a clausal form for the sentence
\(\neg(\forall x P(x) \rightarrow \forall x \exists y Q(x, y)) \lor \neg \forall y P(y)\).
Try to make it as simple as possible.
(b) Consider the following program \(P\):
\[
v = 0; z = 0; \text{while}(! (z == y)) \{z = z + 1; v = v - 1\}; v = v + x
\]
Use weakest preconditions and a suitable invariant to establish
\(\models_p [true] P [v==x-y]\).

Assignment 4 A directed graph consists of a set of nodes connected by directed arcs. Let us assume that the nodes of the graph are named with constants \(\{a, b, \ldots\}\) while the arcs of the graph are represented using a predicate \(A(x, y) = \text{“there is an arc from the node } x \text{ to the node } y\)’.
(a) Define the predicates
\[
C(x, y) = \text{“there is a connection from the node } x \text{ to the node } y\)
\] and \(L(x) = \text{“the graph has a loop that goes through the node } x\)’.
by taking the direction of arcs into account.
(b) Describe the directed graph given below using the predicate \(A\). Use resolution to show that the sentence \(\exists x (L(x) \land C(x, e))\) is a logical consequence of your description and the definitions of predicates \(C\) and \(L\).
\[
a \rightarrow b \rightarrow c
\]