Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 Answer and justify exactly (at most half a page per item).

(a) True or false: it is possible to define the other propositional connectives (¬, ∧, ∨, ↔) using the connectives → and ∨ (exclusive or).

(b) True or false: if Σ₁ and Σ₂ are sets of sentences such that Σ₁ ⊆ Σ₂ and φ is a sentence such that Σ₁ ⊨ φ, then also Σ₂ ⊨ φ.

(c) True or false: a conjunctive normal form φ of a sentence in predicate logic is logically equivalent to the form φ’ obtained from φ by Skolemization.

(d) True or false: the satisfiability problem SAT of propositional logic is NP-complete.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) ⊨ ¬(A ∧ ¬B) ∧ (¬C → A) → (A ∨ B) ∨ (¬A ∧ C)

(b) {∃x ∃y P(x, y), ∀x ∀y (P(x, y) → Q(x, y))} ⊨ ∃x Q(x, x)

(c) {∀x ¬(A(x) ↔ B(x)), ∀y A(y) ∨ ∀y ¬A(y)} ⊨ ∀z B(z) ∨ ∀z ¬B(z)

Tableau proofs must contain all intermediary steps !!!

Assignment 3

(a) Derive a clausal form for the sentence

¬(¬∃y E(y) → ∀y (∃x E(x) → E(y))).

Try to make it as simple as possible.

(b) Consider the following program P:

z = 0; v = x; while(!{z == y}) {z = z + 1; v = v - 1}

Use weakest preconditions and a suitable invariant to establish

⊨ p [true] P [v == x - y].

Assignment 4 Let us represent natural numbers 0, 1, 2,… with ground terms 0, s(0), s(s(0)),… built of a constant symbol 0 and a function symbol s which is interpreted as the function s(x) = x + 1 for natural numbers x.

(a) Let the predicates J₂(x), J₃(x) and J₆(x) mean that a natural number x is divisible by two, three and six, respectively. Use predicate logic to define these predicates such that the definition of the predicate J₆ is based on the definitions of the predicates J₂ and J₃.

(b) Use resolution to show that if a natural number x is divisible by two and three, then the natural number x + 6 is divisible by six.