T-79.144 Logic in Computer Science: Foundations

Examination, May 5, 2004

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 Answer and justify exactly (at most half a page per item).

(a) True or false: the empty clause $\square$ can be obtained from the clauses $\{A, \neg B\}$ and $\{\neg A, B\}$ by resolution.

(b) True or false: if $\models \phi$ and $\phi \wedge \neg\psi$ is unsatisfiable, then $\models \psi$.

(c) True or false: in propositional logic, at most 16 semantically different binary connectives can be defined.

(d) True or false: an existential quantifier can be moved outside an implication as follows: $(\exists x \phi(x) \rightarrow \psi)$ is rewritten as $\forall y(\phi(y) \rightarrow \psi)$ where y is a variable not appearing in formulas $\phi(x)$ and ψ.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models (A \wedge B) \vee (\neg A \wedge C) \rightarrow (A \rightarrow B) \wedge (\neg A \rightarrow C)$

(b) $\models \forall x(P(x) \leftrightarrow \neg Q(x)) \leftrightarrow \neg\exists x(P(x) \leftrightarrow Q(x))$

(c) $\{\forall x(P(x) \rightarrow R(x)), \forall x(\neg Q(x) \rightarrow \neg R(x))\} \models \forall x(P(x) \rightarrow Q(x))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3

(a) Derive a clausal form for the sentence

$$\neg(\forall x \forall y \neg B(y,x) \wedge \exists x(C(x) \rightarrow A(x))).$$

Try to make it as simple as possible.

(b) Consider the following program `P`:

```
z=0;v=x;while(!(z==y)){z=z+1;v=v-1}
```

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\texttt{true}]\ \texttt{P}\ [\texttt{v==x-y}].$$

Assignment 4 Let the predicate $P(x)$ mean that a person x shaves himself, and let the term $f(x)$ refer to the person who is the father of a person x.

(a) Use predicate logic to express the following claim: if a person shaves himself, but his grandfather does not shave himself, then some person shaves himself, but his father does not shave himself.

(b) Use resolution to show that the claim in (a) is valid.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.

Feedback: `http://www.tcs.hut.fi/Studies/T-79.144/feedback.html`