Helsinki University of Technology, Laboratory for Theoretical Computer Science TJ

T-79.144 Logic in Computer Science: Foundations

Examination, December 16, 2003

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 Answer and justify exactly (at most half a page per item).

(a) True or false: the empty clause $\square$ can be obtained from the clauses $\{A, \neg B\}$ and $\{\neg A, B\}$ by resolution.

(b) True or false: $\{P(f(y), g(z, f(f(y)))), P(x, g(x, z))\}$ is unifiable.

(c) True or false: if a set of sentences $\Sigma \subseteq \mathcal{L}$ has exactly one model $\mathcal{A} \subseteq \mathcal{P}$, then it holds for each sentence $\phi \in \mathcal{L}$ that $\Sigma \models \phi$ or $\Sigma \models \neg\phi$.

(d) True or false: the satisfiability problem of propositional logic (SAT) is **NP**-complete.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models (A \land B) \lor (\neg A \land C) \rightarrow (A \rightarrow B) \land (\neg A \rightarrow C)$

(b) $\models \forall x(P(x) \rightarrow Q(x)) \leftrightarrow \neg\exists y(\neg Q(y) \land P(y))$

(c) $\{\forall x \forall y(R(x,y) \rightarrow \neg R(y,y)),\ \exists y R(a,y)\} \models \exists z R(z,z)$

Tableau proofs must contain all intermediary steps !!!

Assignment 3

(a) Derive a clausal form for the sentence

$$\forall x(\exists y R(x,y) \rightarrow P(x)) \rightarrow \exists x Q(x).$$

Try to make it as simple as possible.

(b) Consider the following program `P`:

```
z=0;v=x;while(!(z==y)){z=z+1;v=v-1}
```

Use weakest preconditions and a suitable invariant to establish

$$\models_p [\texttt{true}]\ \texttt{P}\ [\texttt{v==x-y}].$$

Assignment 4 Let us represent natural numbers $0, 1, 2, \ldots$ with ground terms 0, $s(0)$, $s(s(0))$,... built of a constant symbol 0 and a function symbol s which is interpreted as the function $s(x) = x + 1$ for natural numbers x.

(a) Let the predicates $J2(x)$, $J3(x)$ and $J6(x)$ mean that a natural number x is divisible by two, three and six, respectively. Use predicate logic to define these predicates such that the definition of the predicate $J6$ is based on the definitions of the predicates $J2$ and $J3$.

(b) Use resolution to show that if a natural number x is divisible by two and three, then the natural number $x + 6$ is divisible by six.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.

Feedback: `http://www.tcs.hut.fi/Studies/T-79.144/feedback.html`