Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

**Assignment 1** Answer and justify exactly (at most half a page per item).

(a) True or false: if $\Sigma \models \phi$ and $\models \phi \rightarrow \psi$, then $\Sigma \models \psi$.

(b) True or false: the empty clause $\square$ can be obtained from the clauses $\{A, \neg B\}$ and $\{\neg A, B\}$ by resolution.

(c) True or false: Sheffer’s stroke $|$ is definable using Peirce’s arrow $\downarrow$.

(d) True or false: the satisfiability problem of propositional logic is NP-complete.

**Assignment 2** Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models (A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow C) \rightarrow (A \rightarrow B))$

(b) $\models (\forall x (P(x) \rightarrow \neg Q(x))) \rightarrow ((\exists x Q(x)) \rightarrow (\exists x \neg P(x)))$

(c) $\{\forall x (P(x) \rightarrow R(x)), \neg \exists x (\neg R(x) \land Q(x))\} \models \forall x (P(x) \lor Q(x) \rightarrow R(x))$

Tableau proofs must contain all intermediary steps !!!

**Assignment 3**

(a) Derive a clausal form for the sentence $\neg(\forall x \forall y \neg B(y, x) \land \exists x (C(x) \rightarrow A(x)))$. Try to make it as simple as possible.

(b) Use a suitable invariant to establish that the function $\text{min}$ below returns the least integer in a table $a$ for which $\text{size} > 0$ holds.

```c
int min(int a[], int size) {
    int m=a[0], i=1;
    while(i<size) { if(a[i]<m) m=a[i]; i=i+1; }
    return m;
}
```

**Assignment 4** Let us represent natural numbers 0, 1, 2, ... with ground terms $0$, $s(0)$, $s(s(0))$, ... built of a constant symbol 0 and a function symbol $s$ which is interpreted as the function $s(x) = x + 1$ for natural numbers $x$.

(a) Let the predicates $J2(x)$, $J3(x)$ and $J6(x)$ mean that a natural number $x$ is divisible by two, three and six, respectively. Use predicate logic to define these predicates such that the definition of the predicate $J6$ is based on the definitions of the predicates $J2$ and $J3$.

(b) Use resolution to show that if a natural number $x$ is divisible by two and three, then the natural number $x + 6$ is divisible by six.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.