T-79.144 Logic in Computer Science: Foundations

Examination, May 14, 2003

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 Answer and justify exactly (at most half a page per item).

(a) True or false: the empty clause □ can be obtained from the clauses $\{A, \neg B\}$ and $\{\neg A, B\}$ by resolution.

(b) True or false: if ϕ and ψ are two satisfiable sentences, then $\phi \land \psi$ is satisfiable, too.

(c) True or false: if θ and θ' are two most general unifiers of a set of atomic formulas S, then $\theta = \theta'$.

(d) True or false: a proof method M is complete, if every sentence provable by M is valid.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\{B \leftrightarrow \neg C, A \leftrightarrow B \lor C\} \models B \leftrightarrow A \land \neg C$

(b) $\models \forall x(P(x) \land Q(x)) \leftrightarrow \neg(\exists y \neg P(y) \lor \exists z \neg Q(z))$

(c) $\{\forall x \forall y(R(x, y) \rightarrow R(y, x))\} \models \forall x R(a, x)$

Tableau proofs must contain all intermediary steps !!!

Assignment 3

(a) Derive a clausal form for the sentence $\neg(\forall x \forall y \neg B(y, x) \land \exists x(C(x) \rightarrow A(x)))$. Try to make it as simple as possible.

(b) Use a suitable invariant to establish that the function `min` below returns the least integer in a table `a` for which `size` > 0 holds.

```
int min(int a[], int size) {
   int m=a[0], i=1;
   while(i<size) { if(a[i]<m) m=a[i]; i=i+1; }
   return m;
}
```

Assignment 4 Let us represent natural numbers $0, 1, 2, \ldots$ with ground terms 0, $s(0)$, $s(s(0))$,... built of a constant symbol 0 and a function symbol s which is interpreted as the function $s(x) = x + 1$ for natural numbers x.

(a) Let the predicates $J2(x)$, $J3(x)$ and $J6(x)$ mean that a natural number x is divisible by two, three and six, respectively. Use predicate logic to define these predicates such that the definition of the predicate $J6$ is based on the definitions of the predicates $J2$ and $J3$.

(b) Use resolution to show that if a natural number x is divisible by two and three, then the natural number $x + 6$ is divisible by six.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.

Feedback: `http://www.tcs.hut.fi/Studies/T-79.144/feedback.html`