Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

**Assignment 1** Answer and justify exactly (at most half a page per item).

(a) True or false: the empty clause \( \Box \) can be obtained from the clauses \( \{A, \neg B\} \) and \( \{\neg A, B\} \) by resolution.

(b) True or false: if \( \phi \) and \( \psi \) are two satisfiable sentences, then \( \phi \land \psi \) is satisfiable, too.

(c) True or false: if \( \theta \) and \( \theta' \) are two most general unifiers of a set of atomic formulas \( S \), then \( \theta = \theta' \).

(d) True or false: a proof method \( M \) is complete, if every sentence provable by \( M \) is valid.

**Assignment 2** Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) \( \{B \leftrightarrow \neg C, A \leftrightarrow B \lor C\} \models B \leftrightarrow A \land \neg C \)

(b) \( \models \forall x (P(x) \land Q(x)) \leftrightarrow (\exists y \neg P(y) \lor \exists z \neg Q(z)) \)

(c) \( \{\forall x \forall y (R(x, y) \rightarrow R(y, x))\} \models \forall x R(a, x) \)

Tableau proofs must contain all intermediary steps !!!

**Assignment 3**

(a) Derive a clausal form for the sentence \( \neg (\forall x \forall y \neg B(y, x) \land \exists x (C(x) \rightarrow A(x))) \). Try to make it as simple as possible.

(b) Use a suitable invariant to establish that the function \texttt{min} below returns the least integer in a table \( a \) for which \texttt{size} > 0 holds.

```c
int min(int a[], int size) {
    int m=a[0], i=1;
    while(i<size) { if(a[i]<m) m=a[i]; i=i+1; }
    return m;
}
```

**Assignment 4** Let us represent natural numbers 0, 1, 2, ... with ground terms \( 0, s(0), s(s(0)) \) built of a constant symbol 0 and a function symbol \( s \) which is interpreted as the function \( s(x) = x + 1 \) for natural numbers \( x \).

(a) Let the predicates \( J2(x) \), \( J3(x) \) and \( J6(x) \) mean that a natural number \( x \) is divisible by two, three and six, respectively. Use predicate logic to define these predicates such that the definition of the predicate \( J6 \) is based on the definitions of the predicates \( J2 \) and \( J3 \).

(b) Use resolution to show that if a natural number \( x \) is divisible by two and three, then the natural number \( x + 6 \) is divisible by six.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.