Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

**Assignment 1** Answer and justify exactly (at most half a page per item).

(a) True or false: if $\Sigma \vdash \phi$ and $\models \phi \rightarrow \psi$, then $\Sigma \vdash \psi$.

(b) True or false: the empty clause $\Box$ can be obtained from the clauses $\{A, \neg B\}$ and $\{\neg A, B\}$ by resolution.

(c) True or false: Sheffer’s stroke $\uparrow$ is definable using Peirce’s arrow $\downarrow$.

(d) True or false: the satisfiability problem of propositional logic is NP-complete.

**Assignment 2** Examine if the given claim holds using semantic logic. If not, justify by giving a valuation/structure (a counter example).

(a) $\models (A \rightarrow B) \land (\neg A \rightarrow C) \iff (A \land B) \lor (\neg A \land C)$

(b) $\models \forall x \forall y (\exists z (R(x, z) \land R(x, y)) \rightarrow R(x, y))$
   $\rightarrow \forall x \forall y \forall z (R(x, y) \land R(z, y) \rightarrow R(x, z))$

(c) $\models \{ \forall x \forall y (R(x, y) \land R(y, x) \rightarrow R(x, x)),
   \forall x \forall y (R(x, y) \rightarrow R(y, x)) \}$

*Tableau proofs must contain all intermediary steps !!!*

**Assignment 3** Points: 2p for item (a), 3p for item (b) and 1p for item (c).

(a) Derive a clausal form for the sentence $\neg (\forall x \forall y (B(y) \rightarrow A(x, x))) \land
   \exists x (C(x) \rightarrow \forall y A(x, y)))$. Try to make it as simple as possible.

(b) Use a suitable invariant to establish that the function `min` below returns the least integer in a table `a` for which `size > 0` holds.

```c
int min(int a[], int size) {
    int m=a[0], i=1;
    while(i<size) { if(a[i]<m) m=a[i]; i=i+1;}
    return m;
}
```

(c) Explain why a typical PROLOG interpreter is incomplete with respect to SLD resolution using an example.

**Assignment 4** Let us represent lists using a constant symbol $c$ (empty list) and a function symbol $c(\cdot, \cdot)$ so that e.g. $[a, b]$ is represented as $c(a, c(b, e))$. Similarly, let us represent natural numbers 0, 1, 2, ..., as ground terms 0, $s(0)$, $s(s(0))$, ..., consisting of a constant symbol 0 and a function symbol $s(\cdot)$.

(a) Define predicates $L(x, y) = “\text{the length of list } x \text{ is } y”$ and the predicate $P(x, y) = “\text{list } x \text{ is a prefix of list } y”$ using sentences of predicate logic so that all lists are covered.

(b) Show that $\exists x (P(x, c(a, c(b, c(b, c(a, e))))) \land L(x, s(s(0))))$ is a logical consequence of your definitions using semantic tableaux.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.