# T-79.144 Logic in Computer Science: Foundations

Examination, December 17, 2002

**Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!**

**Assignment 1** Answer and justify exactly (at most half a page per item).

(a) True or false: if $\Sigma \models \phi$ and $\models \phi \to \psi$, then $\Sigma \models \psi$.

(b) True or false: the empty clause $\Box$ can be obtained from the clauses $\{A, \neg B\}$ and $\{\neg A, B\}$ by resolution.

(c) True or false: Sheffer's stroke $|$ is definable using Peirce's arrow $\downarrow$.

(d) True or false: the satisfiability problem of propositional logic is **NP**-complete.

**Assignment 2** Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models (A \to B) \wedge (\neg A \to C) \leftrightarrow (A \wedge B) \vee (\neg A \wedge C)$

(b) $\models \forall x \forall y (\exists z (R(x,z) \wedge R(z,y)) \to R(x,y))$
$\to \forall z \forall y \forall x (R(x,y) \wedge R(y,z) \to R(x,z))$

(c) $\{ \ \forall x \forall y (R(x,y) \wedge R(y,x) \to R(x,x)),$
$\forall x \forall y (R(x,y) \to R(y,x)) \ \} \models \forall x \forall y \, R(x,y)$

Tableau proofs must contain all intermediary steps !!!

**Assignment 3** Points: $2p$ for item (a), $3p$ for item (b) and $1p$ for item (c).

(a) Derive a clausal form for the sentence $\neg(\forall x \forall y (B(y) \to A(x,x)) \wedge \exists x (C(x) \to \forall y A(x,y)))$. Try to make it as simple as possible.

(b) Use a suitable invariant to establish that the function `min` below returns the least integer in a table `a` for which `size` $> 0$ holds.

```
int min(int a[], int size) {
   int m=a[0], i=1;
   while(i<size) { if(a[i]<m) m=a[i]; i=i+1;}
   return m;
}
```

(c) Explain why a typical PROLOG interpreter is incomplete with respect to SLD resolution using an example.

**Assignment 4** Let us represent lists using a constant symbol $e$ (empty list) and a function symbol $c(\cdot,\cdot)$ so that e.g. $[a,b]$ is represented as $c(a,c(b,e))$. Similarly, let us represent natural numbers $0,1,2,\ldots$ as ground terms $0, s(0), s(s(0)),\ldots$ consisting of a constant symbol 0 and a function symbol $s(\cdot)$.

(a) Define predicates $L(x,y)$ = "the length of list $x$ is $y$" and the predicate $P(x,y)$ = "list $x$ is a prefix of list $y$" using sentences of predicate logic so that all lists are covered.

(b) Show that $\exists x ( P(x, c(a,c(b,c(b,c(a,e))))) \wedge L(x, s(s(0))) )$ is a a logical consequence of your definitions using semantic tableaux.

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The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.

Feedback: `http://www.tcs.hut.fi/Studies/T-79.144/feedback.html`