Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Helsinki University of Technology, Laboratory for Theoretical Computer Science  TJ T-79.144 Logic in Computer Science: Foundations
Examination, September 3, 2002

Assignment 1 Answer and justify briefly, but exactly.
(a) Does the following hold: Sheffer’s stroke \( | \) is definable in terms of Peirce’s arrow \( \downarrow \).
(b) Does the following hold: if \( \phi \) and \( \psi \) are different sentences in propositional logic, then their clausal forms are different, too.
(c) Does the following hold: if \( \Sigma \not\models \phi \) and \( \Gamma \subseteq \Sigma \), then \( \Gamma \not\models \phi \) (\( \Sigma \) and \( \Gamma \) are sets of sentences).
(d) Does the following hold: predicate logic is decidable.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).
(a) \( \models (\neg B \rightarrow \neg A) \rightarrow ((\neg B \rightarrow A) \rightarrow B) \)
(b) \( \{ \forall x \forall y (R(x, y) \rightarrow R(y, x)) \} \models \forall x \forall y (R(x, y) \lor R(y, x)) \)
(c) \( \models \forall x (P(x) \leftrightarrow \neg Q(x)) \leftrightarrow \exists x (P(x) \leftrightarrow Q(x)) \)
Tableau proofs must contain all intermediary steps !!!

Assignment 3 Formalize the following claims in terms of predicate logic:
1. If a brick is on another brick, it is not on the table.
2. Every brick is on the table or on another brick.
3. No brick is on a brick which is also on some other brick.
4. If a brick is on another brick, then the latter brick is on the table.

Use resolution to show that the sentence 4 is a logical consequence of the sentences 1-3.

Assignment 4 A directed graph consists of a set of nodes connected by directed arcs. Assume that nodes are represented with constants \( \{ a, b, \ldots \} \) while arcs are represented with a binary predicate \( A(x, y) = \text{“there is an arc leading from the node } x \text{ to the node } y \text{”} \).

(a) Define the predicates

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C(x, y) = \text{“there is a connection from the node } x \text{ to the node } y \text{”}
\]
and \( L(x) = \text{“the graph has a loop that goes through the node } x \text{”} \)

by taking the direction of arcs into account.

(b) Describe the directed graph below using the predicate \( A \). Show that \( \exists x \forall y (L(x) \land A(x, y) \land L(y)) \) is a logical consequence of your description and the definitions of predicates \( C \) and \( L \). Use semantic tableaux.

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\begin{array}{c}
\text{a} \\
\text{b} \\
\text{c}
\end{array}
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The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.