Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Helsinki University of Technology, Laboratory for Theoretical Computer Science TJ

T-79.144 Logic in Computer Science: Foundations

Examination, January 8, 2002

Assignment 1 Answer and justify briefly, but exactly.

(a) Does the following hold: the empty clause $\Box$ can be obtained from the clauses $\{A, \neg B\}$ and $\{\neg A, B\}$ by resolution.

(b) Does the folloing hold: if $\Sigma \models \phi$ and $\Sigma \models \neg\phi$ for some sentence ϕ, then the set of sentences Σ is unsatisfiable.

(c) Does the following hold: predicate logic is decidable.

(d) Does the following hold: a propositional sentence ϕ has at most as many *subsentences* as it has atomic sentences and connectives ($\neg$,$\wedge$,$\vee$,$\rightarrow$,$\leftrightarrow$).

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models (A \rightarrow (B \vee C)) \rightarrow (\neg B \rightarrow (\neg C \rightarrow \neg A))$

(b) $\models \forall x \exists y R(x, y) \rightarrow (\forall y(\neg S(y) \rightarrow \neg \exists x R(x, y)) \rightarrow \exists x S(x))$

(c) $\{\forall x \exists y(P(x) \rightarrow Q(y)), \forall x P(x)\} \models \forall y Q(y)$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 Let a ternary predicate parents(x, y, z) mean that the parents of a person x are y and z. Using this predicate, define the binary predicate relative(x, y) which means that x is a relative of y. Give a resolution proof that Kerttu is a relative of Kustaa using the following database in addition to your definition.

$$\begin{array}{l} \text{parents(kerttu, jaakoppi, hanna)} \\ \text{parents(jaakoppi, reino, lahja)} \\ \text{parents(kustaa, salme, reino)} \end{array}$$

Hint: relatives have an ancestor in common!

Assignment 4 Consider a binary predicate R which is interpreted as a binary relation $R^{\mathcal{A}} \subseteq A \times A$ with respect to a universe A.

(a) Give sentences of predicate logic that define when $R^{\mathcal{A}}$ is (1) *reflexive*, (2) *symmetric*, (3) *transitive* and (4) an *equivalence relation*.

(b) Use semantic tableaux to establish that $R^{\mathcal{A}}$ is an equivalence relation, if it is symmetric, transitive and *serial* (as defined by $\forall x \exists y R(x, y)$).

(c) Examine if an equivalence relation is always serial. Again, use semantic tableaux.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.