Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Helsinki University of Technology, Laboratory for Theoretical Computer Science TJ T-79.144 Logic in Computer Science: Foundations
Examination, January 8, 2002

Assignment 1 Answer and justify briefly, but exactly.

(a) Does the following hold: the empty clause $\square$ can be obtained from the clauses \{A, $\neg$B\} and \{$\neg$A, B\} by resolution.

(b) Does the following hold: if $\Sigma \models \phi$ and $\Sigma \models \neg\phi$ for some sentence $\phi$, then the set of sentences $\Sigma$ is unsatisfiable.

(c) Does the following hold: predicate logic is decidable.

(d) Does the following hold: a propositional sentence $\phi$ has at most as many subsentences as it has atomic sentences and connectives ($\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$).

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models (A \rightarrow (B \lor C)) \rightarrow (\neg B \rightarrow (\neg C \rightarrow \neg A))$

(b) $\models \forall x \exists y R(x, y) \rightarrow (\exists y (\neg S(y) \rightarrow \neg \exists x R(x, y))) \rightarrow \exists x S(x))$

(c) \{$\forall x \exists y (P(x) \rightarrow Q(y)), \forall x P(x)$\} $\models \forall y Q(y)$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 Let a ternary predicate parents(x, y, z) mean that the parents of a person x are y and z. Using this predicate, define the binary predicate relative(x, y) which means that x is a relative of y. Give a resolution proof that Kerntu is a relative of Kustaa using the following database in addition to your definition.

parents(kerntu, jaakoppi, hanna)
parents(jaakoppi, reino, lahja)
parents(kustaa, salme, reino)

Hint: relatives have an ancestor in common!

Assignment 4 Consider a binary predicate R which is interpreted as a binary relation $R^A \subseteq A \times A$ with respect to a universe A.

(a) Give sentences of predicate logic that define when $R^A$ is (1) reflexive, (2) symmetric, (3) transitive and (4) an equivalence relation.

(b) Use semantic tableaux to establish that $R^A$ is an equivalence relation, if it is symmetric, transitive and serial (as defined by $\forall x \exists y R(x, y)$).

(c) Examine if an equivalence relation is always serial. Again, use semantic tableaux.

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The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.