Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 Answer and justify briefly, but exactly.
(a) Does the following hold: it is possible to define the other propositional connectives (\&, \lor, \leftrightarrow) using the connectives \rightarrow and \lor (exclusive or).
(b) Does the following hold: if \Sigma_1 and \Sigma_2 are sets of sentences such that \Sigma_1 \subseteq \Sigma_2 and \phi is a sentence such that \Sigma_1 \models \phi, then also \Sigma_2 \models \phi.
(c) Does the following hold: a conjunctive normal form \phi of a sentence in predicate logic is logically equivalent to the form \phi' obtained from \phi by Skolemization.
(d) Does the following hold: the satisfiability problem SAT of propositional logic is NP-complete.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).
(a) \models (\neg C \rightarrow A \land \neg B) \rightarrow ((\neg C \rightarrow (A \rightarrow B)) \rightarrow C).
(b) \{\exists x \exists y P(x, y), \forall x \forall y (P(x, y) \rightarrow Q(x, y))\} \models \exists x Q(x, x)
(c) \{\forall x (P(x) \rightarrow R(x)), \forall x (\neg Q(x) \rightarrow \neg R(x))\} \models \forall x (P(x) \rightarrow Q(x))

Tableau proofs must contain all intermediary steps !!!

Assignment 3 Binary trees are represented in terms of a binary function symbol \text{i} (inner nodes) and a unary function symbol \text{l} (leaf nodes). In this way, the binary tree in the picture gets a representation \text{i}(\text{i}(\text{l}(c), \text{l}(a)), \text{l}(b)).
(a) Let the predicate \text{M}(x, y) mean that binary tree \text{x} is the mirror image of binary tree \text{y}. Define the predicate \text{M} using sentences of predicate logic (assuming the representation given above).
(b) Use resolution to find the mirror image of the following binary tree:

Assignment 4 Let us represent natural numbers 0, 1, 2, \ldots with ground terms 0, s(0), s(s(0)), \ldots built of a constant symbol 0 and a function symbol \text{s} which is interpreted as the function \text{s}(x) = x + 1 for natural numbers \text{x}.
(a) Let the predicates \text{J}2(x), \text{J}3(x) and \text{J}6(x) mean that a natural number \text{x} is divisible by two, three and six, respectively. Use predicate logic to define these predicates such that the definition of the predicate \text{J}6 is based on the definitions of the predicates \text{J}2 and \text{J}3.
(b) Use semantic tableaux to show that if a natural number \text{x} is divisible by two and three, then the natural number \text{x} + 6 is divisible by six.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.