Helsinki University of Technology, Laboratory for Theoretical Computer Science TJ

Tik-79.144 Logic in computer science: foundations

Examination, December 18, 2001

Please note the following: your answers will be graded only if you have passed all the three home assignments before the exam!

Assignment 1 Answer and justify briefly, but exactly.

(a) Does the following hold: it is possible to define the other propositional connectives ($\wedge$,$\vee$,$\leftrightarrow$) using the connectives $\rightarrow$ and $\veebar$ (exclusive or).

(b) Does the folloing hold: if Σ_1 and Σ_2 are sets of sentences such that $\Sigma_1 \subseteq \Sigma_2$ and ϕ is a sentence such that $\Sigma_1 \models \phi$, then also $\Sigma_2 \models \phi$.

(c) Does the following hold: a conjunctive normal form ϕ of a sentence in predicate logic is logically equivalent to the form ϕ' obtained from ϕ by Skolemization.

(d) Does the following hold: the satisfiability problem SAT of propositional logic is **NP**-complete.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models (\neg C \rightarrow A \wedge \neg B) \rightarrow ((\neg C \rightarrow (A \rightarrow B)) \rightarrow C)$.

(b) $\{\exists x \exists y P(x,y), \forall x \forall y (P(x,y) \rightarrow Q(x,y))\} \models \exists x Q(x,x)$

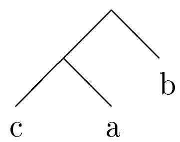
(c) $\{\forall x(P(x) \rightarrow R(x)), \forall x(\neg Q(x) \rightarrow \neg R(x))\} \models \forall x(P(x) \rightarrow Q(x))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 Binary trees are represented in terms of a binary function symbol i (inner nodes) and a unary function symbol l (leaf nodes). In this way, the binary tree in the picture gets a representation $i(i(l(c), l(a)), l(b))$.

(a) Let the predicate $\mathrm{M}(x,y)$ mean that binary tree x is the mirror image of binary tree y. Define the predicate M using sentences of predicate logic (assuming the representation given above).

(b) Use resolution to find the mirror image of the following binary tree:



Assignment 4 Let us represent natural numbers $0, 1, 2, \ldots$ with ground terms 0, $s(0)$, $s(s(0))$,... built of a constant symbol 0 and a function symbol s which is interpreted as the function $s(x) = x + 1$ for natural numbers x.

(a) Let the predicates $J2(x)$, $J3(x)$ and $J6(x)$ mean that a natural number x is divisible by two, three and six, respectively. Use predicate logic to define these predicates such that the definition of the predicate $J6$ is based on the definitions of the predicates $J2$ and $J3$.

(b) Use semantic tableaux to show that if a natural number x is divisible by two and three, then the natural number $x + 6$ is divisible by six.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.