Tik-79.144 Logic in computer science: foundations

Examination, September 4, 2001

Assignment 1 Answer and justify briefly, but exactly.

(a) Does the following hold: it holds for every set of sentences Σ and every sentence ϕ that if $\Sigma \models \neg\phi$, then $\Sigma \cup \{\phi\}$ is unsatisfiable.

(b) Does the following hold: at most 16 different binary connectives can be defined for propositional logic.

(c) Does the following hold: propositional logic is decidable.

(d) Does the following hold: the empty clause $\square$ can be obtained from the clauses $\{P(x), P(y)\}$ and $\{\neg P(z), \neg P(w)\}$ by resolution.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models (A \to (B \to C)) \to ((A \to B) \to (A \to C))$

(b) $\{\forall x \exists y (P(x) \to Q(y)), \forall x P(x)\} \models \forall y Q(y)$

(c) $\{\forall x \forall y \forall z (R(x,y) \land R(y,z) \to R(x,z)), R(a,b), R(b,a)\} \models R(a,a)$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 Show that the sentence

$$\exists x (R(x) \land \neg R(f(f(x)))) \to \exists x (R(x) \land \neg R(f(x)))$$

is valid using resolution.

Assignment 4 Let us represent natural numbers $0, 1, 2, \ldots$ with ground terms 0, $s(0)$, $s(s(0))$,... built of a constant symbol 0 and a function symbol s which is interpreted as the function $s(x) = x + 1$ for natural numbers x.

(a) Let the predicates $J2(x)$, $J3(x)$ and $J6(x)$ mean that a natural number x is divisible by two, three and six, respectively. Use predicate logic to define these predicates such that the definition of the predicate $J6$ is based on the definitions of the predicates $J2$ and $J3$.

(b) Use semantic tableaux to show that if a natural number x is divisible by two and three, then the natural number $x + 6$ is divisible by six.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.