Helsinki University of Technology, Laboratory for Theoretical Computer Science  
TJ Tik-79.144 Logic in computer science: foundations  
Examination, September 4, 2001

Assignment 1 Answer and justify briefly, but exactly.

(a) Does the following hold: it holds for every set of sentences \( \Sigma \) and every sentence \( \phi \) that if \( \Sigma \models \neg \phi \), then \( \Sigma \cup \{ \phi \} \) is unsatisfiable.

(b) Does the following hold: at most 16 different binary connectives can be defined for propositional logic.

(c) Does the following hold: propositional logic is decidable.

(d) Does the following hold: the empty clause \( \Box \) can be obtained from the clauses \( \{ P(x), P(y) \} \) and \( \{ \neg P(x), \neg P(w) \} \) by resolution.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) \( \models (A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow C)) \)

(b) \( \{ \forall x \exists y (P(x) \rightarrow Q(y)), \forall x P(x) \} \models \forall y Q(y) \)

(c) \( \{ \forall x \forall y \forall z (R(x, y) \land R(y, z) \rightarrow R(x, z)), R(a, b), R(b, a) \} \models R(a, a) \)

Tableau proofs must contain all intermediary steps !!!

Assignment 3 Show that the sentence

\[ \exists x (R(x) \land \neg R(f(f(x)))) \rightarrow \exists x (R(x) \land \neg R(f(x))) \]

is valid using resolution.

Assignment 4 Let us represent natural numbers 0, 1, 2, . . . with ground terms 0, s(0), s(s(0)), . . . built of a constant symbol 0 and a function symbol \( \text{s} \) which is interpreted as the function \( s(x) = x + 1 \) for natural numbers \( x \).

(a) Let the predicates \( J2(x) \), \( J3(x) \) and \( J6(x) \) mean that a natural number \( x \) is divisible by two, three and six, respectively. Use predicate logic to define these predicates such that the definition of the predicate \( J6 \) is based on the definitions of the predicates \( J2 \) and \( J3 \).

(b) Use semantic tableaux to show that if a natural number \( x \) is divisible by two and three, then the natural number \( x + 6 \) is divisible by six.

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The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.