Tik-79.144 Logic in computer science: foundations

Examination, May 10, 2001

Assignment 1 Answer and justify briefly, but exactly.

(a) Does the following hold: if $\mathcal{V}_1$ and $\mathcal{V}_2$ are valuations such that $\mathcal{V}_1(A) = \mathcal{V}_2(A)$ for all atomic sentences A in a sentence ϕ, then $\mathcal{V}_1(\phi) = \mathcal{V}_2(\phi)$.

(b) Does the following hold: predicate logic is semidecidable.

(c) Does the following hold: if $\Sigma \models \phi$ and $\models \phi \to \psi$, then $\Sigma \models \psi$.

(d) Does the following hold: the empty clause $\Box$ can be obtained from the clauses $\{A, \neg B\}$ and $\{\neg A, B\}$ by resolution.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\{B \leftrightarrow \neg C, A \leftrightarrow B \vee C\} \models B \leftrightarrow A \wedge \neg C$

(b) $\{\forall x \exists y (P(x) \to Q(y)), \forall x P(x)\} \models \forall y Q(y)$

(c) $\{\forall x \forall y \forall z (R(x,y) \wedge R(y,z) \to R(x,z)), R(a,b)\} \models \neg R(b,a)$.

Tableau proofs must contain all intermediary steps !!!

Assignment 3 Formalize the following claims in terms of predicate logic:

1. If a brick is on another brick, it is not on the table.
2. Every brick is on the table or on another brick.
3. No brick is on a brick which is also on some other brick.

---

4. If a brick is on another brick, then the latter brick is on the table.

Use resolution to show that the sentence 4 is a logical consequence of the sentences 1-3.

Assignment 4 Let the predicate $P(x)$ mean that a person x shaves himself, and let the term $f(x)$ refer to the father of a person x.

(a) Use predicate logic to express the following claim: if a person shaves himself, but his grandfather does not shave himself, then some person shaves himself, but his father does not shave himself.

(b) Use semantic tableaux to show that the claim in (a) is valid.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.