Assignment 1 Answer and justify briefly, but exactly.

(a) Does the following hold: propositional logic is decidable.

(b) Does the following hold: the empty clause □ can be obtained from the
classes \{P(x, y), P(y, x)\} and \{¬P(z, z), ¬P(w, w)\} by resolution.

(c) Does the following hold: if a set of sentences \(\Sigma\) has at most one model,
then it holds for each sentence \(\phi\) that \(\Sigma \models \phi\) or \(\Sigma \models \neg \phi\).

(d) Does the following hold: a sentence \(\phi\) has at most as many subsentences
as it has atomic sentences and connectives \(\neg, \wedge, \vee, \to, \leftrightarrow\).

Assignment 2 Examine if the given claim holds using semantic tableaux.
If not, justify by giving a valuation/structure (a counter example).

(a) \(\models (\neg B \to \neg A) \to ((\neg B \to A) \to B)\)

(b) \(\{\forall x \exists y (P(x) \to Q(y)), \forall x P(x)\} \equiv \forall y Q(y)\)

(c) \(\{\forall x (A(x) \leftrightarrow \neg B(x)), \forall x (B(x) \leftrightarrow \neg C(x)), \forall x (C(x) \leftrightarrow \neg A(x))\} = \forall x (A(x) \land B(x) \land C(x))\)

Tableau proofs must contain all intermediary steps !!!

Assignment 3 Show that the sentence

\[\exists x R(x) \land \neg R(f(f(x))) \to \exists x R(x) \land \neg R(f(x))\]

is valid by linear resolution.

Assignment 4 Let the predicate \(M(x, y)\) mean that a person \(x\) has met a
person \(y\), and the predicate \(S(x, y)\) that a person \(x\) is sick of an infectious
disease \(y\).

(a) Use the predicates given above to define a predicate \(D(x, y)\) which
means that a person \(x\) is in danger of infection by a disease \(y\).

(b) Consider the following database.

\[M(\text{valentine, ronald}) \quad S(\text{valentine, flu})\]

\[M(\text{harry, chickenpox}) \lor S(\text{ronald, rubeola})\]

\[M(\text{harry, lillian})\]

Use semantic tableaux to show that somebody is in danger of infection
by flu and chickenpox, or by flu and rubeola. The tableaux proof must
be based on the database and the definition of the predicate \(D(x, y)\).

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The name of the course, the course code, the date, your name, your
student id, and your signature must appear on every sheet of your answers.