Tik-79.144 Logic in computer science: foundations

Examination, December 19, 2000

Assignment 1 Answer and justify briefly, but exactly.

(a) Does the following hold: at most 16 different binary connectives can be defined for propositional logic.

(b) Does the following hold: the set $\{P(x, f(x, z)), P(h(y), f(z, y))\}$ is unifiable.

(c) Does the following hold: predicate logic is decidable.

(d) Does the following hold: if a set of sentences Σ has exactly one model, then it holds for each sentence ϕ that $\Sigma \models \phi$ or $\Sigma \models \neg\phi$ (exclusively).

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\{B \leftrightarrow \neg C, A \leftrightarrow B \vee C\} \models B \leftrightarrow A \wedge \neg C$

(b) $\{\forall x(P(x) \rightarrow R(x)), \forall x(\neg Q(x) \rightarrow \neg R(x))\} \models \forall x(P(x) \rightarrow Q(x))$

(c) $\{\exists x \exists y P(x, y), \forall x \forall y(P(x, y) \rightarrow Q(x, y))\} \models \exists x Q(x, x)$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 The quantifier $\exists! x$ means that "there is only one x". The claim $\exists! x\, \phi(x)$ can be expressed in predicate logic as the sentence

$$(\exists x \phi(x)) \wedge (\forall x \forall y(\phi(x) \wedge \phi(y) \rightarrow x = y)).$$

Formalize the following sentences in predicate logic:

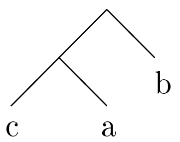
1. There is only one white-bearded.
2. Every Santa Claus is white-bearded.
3. Every white-bearded is Santa Claus.

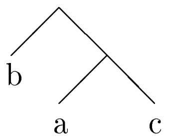
---

4. There is only one Santa Claus.

Give a resolution proof which shows that the sentence 4 is a logical consequence of the sentences 1-3.

Assignment 4 Binary trees are represented in terms of a binary function symbol i (inner nodes) and a unary function symbol l (leaf nodes). In this way, the upper tree in the picture gets a representation $i(i(l(c), l(a)), l(b))$.





(a) Let the predicate $\mathrm{M}(x, y)$ mean that binary tree x is the mirror image of binary tree y. Define the predicate M using sentences of predicate logic such that you can infer whether any given two binary trees are mirror images of each other (assuming the representation given above).

(b) Use semantic tableaux to show that the upper binary tree is the mirror image of the lower binary tree.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.