Assignment 1 Answer and justify briefly, but exactly.

(a) Does the following hold: if $\Sigma \models \phi$, then $\Sigma \cup \{\neg \phi\}$ is unsatisfiable.

(b) Does the following hold: if $\theta$ and $\theta'$ are two most general unifiers of a set of atomic formulas $S$, then $\theta = \theta'$.

(c) Does the following hold: at most 16 different binary connectives can be defined for propositional logic.

(d) Does the following hold: the empty clause $\Box$ can be obtained from the clauses $\{P(x), P(y)\}$ and $\{\neg P(z), \neg P(w)\}$ by resolution.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $| (\neg B \rightarrow \neg A) \rightarrow ((\neg B \rightarrow A) \rightarrow B) |

(b) $\{\forall x \exists y(P(x) \rightarrow Q(y)), \forall x P(x)\} = \forall y Q(y)$

(c) $\{\forall x(A(x) \leftrightarrow \neg B(x)), \forall x (B(x) \leftrightarrow \neg C(x)), \forall x (C(x) \leftrightarrow \neg A(x))\} = \forall x (A(x) \land B(x) \land C(x))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 Show that the sentence

$$\exists x (R(x) \land \neg R(f(f(x)))) \rightarrow \exists x (R(x) \land \neg R(f(x)))$$

is valid by linear resolution.

Assignment 4 Natural numbers $0, 1, 2, \ldots$ are represented as ground terms $0, s(0), s(s(0)), \ldots$ built of a constant symbol $0$ and a function symbol $s$ which is interpreted as the function $s(x) = x + 1$ for natural numbers $x$.

(a) Let the predicates $J2(x), J3(x)$ and $J6(x)$ mean that a natural number $x$ is divisible by two, three and six, respectively. Use predicate logic to define these predicates such that the definition of the predicate $J6$ is based on the definitions of the predicates $J2$ and $J3$.

(b) Use semantic tableaux to show that if a natural number $n$ is divisible by two and three, then the natural number $n + 6$ is divisible by six.