Tik-79.144 Logic in computer science: foundations

Examination, May 12, 2000

Assignment 1 Answer and justify briefly, but exactly.

(a) Does the following hold: if $\Sigma \models \phi$, then $\Sigma \cup \{\neg\phi\}$ is unsatisfiable.

(b) Does the following hold: if θ and θ' are two most general unifiers of a set of atomic formulas S, then $\theta = \theta'$.

(c) Does the following hold: at most 16 different binary connectives can be defined for propositional logic.

(d) Does the following hold: the empty clause $\Box$ can be obtained from the clauses $\{P(x), P(y)\}$ and $\{\neg P(z), \neg P(w)\}$ by resolution.

Assignment 2 Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

(a) $\models (\neg B \rightarrow \neg A) \rightarrow ((\neg B \rightarrow A) \rightarrow B)$

(b) $\{\forall x \exists y (P(x) \rightarrow Q(y)), \forall x P(x)\} \models \forall y Q(y)$

(c) $\{\forall x (A(x) \leftrightarrow \neg B(x)), \forall x (B(x) \leftrightarrow \neg C(x)), \forall x (C(x) \leftrightarrow \neg A(x))\} \models \forall x (A(x) \wedge B(x) \wedge C(x))$

Tableau proofs must contain all intermediary steps !!!

Assignment 3 Show that the sentence

$$\exists x (R(x) \wedge \neg R(f(f(x)))) \rightarrow \exists x (R(x) \wedge \neg R(f(x)))$$

is valid by linear resolution.

Assignment 4 Natural numbers $0, 1, 2, \ldots$ are represented as ground terms 0, $s(0)$, $s(s(0))$,... built of a constant symbol 0 and a function symbol s which is interpreted as the function $s(x) = x + 1$ for natural numbers x.

(a) Let the predicates $J2(x)$, $J3(x)$ and $J6(x)$ mean that a natural number x is divisible by two, three and six, respectively. Use predicate logic to define these predicates such that the definition of the predicate $J6$ is based on the definitions of the predicates $J2$ and $J3$.

(b) Use semantic tableaux to show that if a natural number n is divisible by two and three, then the natural number $n + 6$ is divisible by six.

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.