Helsinki University of Technology, Laboratory for Theoretical Computer Science TJ

# Tik-79.144 Logic in computer science: foundations

Examination, January 14, 2000

**Assignment 1** Answer and justify briefly, but exactly.

(a) Does the following hold: the sentence $A \wedge B \wedge C$ is a disjunctive normal form of the sentence $\neg(A \rightarrow (\neg B \vee \neg C))$.

(b) Does the following hold: if $\models \phi \vee \psi$, then $\models \phi$ or $\models \psi$.

(c) Does the following hold: the satisfiability problem of propositional logic is **NP**-complete.

(d) Does the following hold: the empty clause $\Box$ can be obtained from the clauses $\{A, \neg B\}$ and $\{\neg A, B\}$ by resolution.

**Assignment 2** Examine if the given claim holds using semantic tableaux. If not, justify by giving a valuation/structure (a counter example).

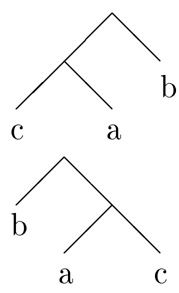
(a) $\{A \rightarrow D, A \vee \neg B \vee C, D \rightarrow C, B\} \models D$

(b) $\models \forall x \exists y R(x,y) \rightarrow (\forall y(\neg S(y) \rightarrow \neg \exists x R(x,y)) \rightarrow \exists x S(x))$

(c) $\{\forall x(P(x) \rightarrow Q(x) \vee R(x)), \neg \exists x R(x)\} \models \forall x(\neg Q(x) \rightarrow \neg P(x))$

Tableau proofs must contain all intermediary steps !!!

**Assignment 3** Binary trees are represented in terms of a binary function symbol $i$ (inner nodes) and a unary function symbol $l$ (leaf nodes). In this way, the upper tree in the picture gets a representation $i(i(l(c), l(a)), l(b))$.




(a) Let the predicate $\mathrm{M}(x,y)$ mean that binary tree $x$ is the mirror image of binary tree $y$. Define the predicate M using sentences of predicate logic such that you can infer whether any given two binary trees are mirror images of each other (assuming the representation given above).

(b) Use resolution to show that the upper binary tree is the mirror image of the lower binary tree.

**Assignment 4** Natural numbers $0, 1, 2, \ldots$ are represented as ground terms $0$, $s(0)$, $s(s(0))$,... built of a constant symbol 0 and a function symbol $s$ which is interpreted as the function $s(x) = x + 1$ for natural numbers $x$.

(a) Give sentences of predicate logic to define predicates $O(x) =$ "$x$ is odd", $E(x) =$ "$x$ is even" and $G(x,y) =$ "$x$ is greater than $y$" for all natural numbers $x$ and $y$.

(b) Use semantic tableaux to prove that there exists an even natural number which is greater than an odd natural number.

---

The name of the course, the course code, the date, your name, your student id, and your signature must appear on every sheet of your answers.