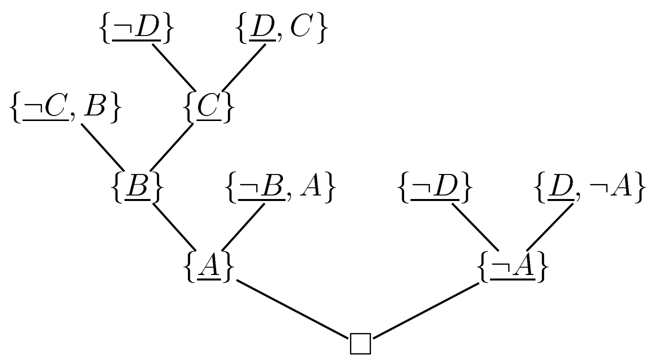
Tik-79.144 **Syksy 2000**
Logic in Computer Science: Foundations
Exercises 5
Solutions

1. First transform the sentences to the clausul form:

Lause		Klausuulit
$\neg A \to \neg B$		$\{A, \neg B\}$
$\neg B \to \neg C$		$\{B, \neg C\}$
$(C \to A) \to D$	$\equiv \neg(\neg C \vee A) \vee D$	
	$\equiv (C \wedge \neg A) \vee D$	
	$\equiv (C \vee D) \wedge (\neg A \vee D)$	$\{C, D\}, \{\neg A, D\}$
$\neg D$		$\{\neg D\}$

Next, do the resolution proof:



2. Transform the propositions to conjunctive normal form and to a set of clauses. The last item of the table is the negation of the proposition to prove: $P1 \wedge P2$.

Proposition		Clauses
$Pi \vee Ki \vee Vi$		$\{Pi, Ki, Vi\}$
$Pi \to \neg Ki \wedge \neg Vi$	$\equiv \neg Pi \vee (\neg Ki \wedge \neg Vi)$	
	$\equiv (\neg Pi \vee \neg Ki) \wedge (\neg Pi \vee \neg Vi)$	$\{\neg Pi, \neg Ki\}, \{\neg Pi, \neg Vi\}$
$Ki \to \neg Pi \wedge \neg Vi$	$\equiv (\neg Ki \vee \neg Pi) \wedge (\neg Ki \vee \neg Vi)$	$\{\neg Pi, \neg Ki\}, \{\neg Ki, \neg Vi\}$
$Vi \to \neg Pi \wedge \neg Ki$	$\equiv (\neg Vi \vee \neg Pi) \wedge (\neg Vi \vee \neg Ki)$	$\{\neg Pi, \neg Vi\}, \{\neg Ki, \neg Vi\}$
$\neg(V1 \wedge V2)$	$\equiv \neg V1 \vee \neg V2$	$\{\neg V1, \neg V2\}$
$P1 \to (K2 \vee V2)$	$\equiv \neg P1 \vee K2 \vee V2$	$\{\neg P1, K2, V2\}$
$P2 \to (K1 \vee V1)$	$\equiv \neg P2 \vee K1 \vee V1$	$\{\neg P2, K1, V1\}$
$P1 \wedge P2$		$\{P1\}, \{P2\}$

The idea of the resolution proofs is to show that the above set of clauses is unsatisfiable, and so our supposition that it is possible to have both lights $P1$ and $P2$ on at the same time must be false. This is achieved by deriving the empty clause $\{\}$ from the set of clause.

$$\begin{tikzcd}[column sep=tiny]
\{\neg P1, K2, V2\} \arrow[dr, dash] & & \{P1\} \arrow[dl, dash] & & & & \\
& \{K2, V2\} \arrow[dr, dash] & & \{\neg P2, \neg K2\} \arrow[dl, dash] & & & \\
& & \{\neg P2, V2\} \arrow[dr, dash] & & \{\neg P2, \neg V2\} \arrow[dl, dash] & & \\
& & & \{\neg P2\} \arrow[dr, dash] & & \{P2\} \arrow[dl, dash] & \\
& & & & \{\} & &
\end{tikzcd}$$

3. The chemical reactions can be formalized as implications, which can then be transformed into clausul form. The resulting clauses are:

(1)

$$\begin{aligned}
&\mathrm{MgO} + \mathrm{H_2} \to \mathrm{Mg} + \mathrm{H_2O} \\
\Longrightarrow &\mathrm{MgO} \wedge \mathrm{H_2} \to \mathrm{Mg} \wedge \mathrm{H_2O} \\
\Longrightarrow &\neg\mathrm{MgO} \vee \neg\mathrm{H_2} \vee (\mathrm{Mg} \wedge \mathrm{H_2O}) \\
\Longrightarrow &(\neg\mathrm{MgO} \vee \neg\mathrm{H_2} \vee \mathrm{Mg}) \wedge (\neg\mathrm{MgO} \vee \neg\mathrm{H_2} \vee \mathrm{H_2O})
\end{aligned}$$

The first reaction results in two clauses: : $\{\neg\mathrm{MgO}, \neg\mathrm{H_2}, \mathrm{Mg}\}$ and $\{\neg\mathrm{MgO}, \neg\mathrm{H_2}, \mathrm{H_2O}\}$.

(2)

$$\begin{aligned}
&\mathrm{C} + \mathrm{O_2} \to \mathrm{CO_2} \\
\Longrightarrow &\mathrm{C} \wedge \mathrm{O_2} \to \mathrm{CO_2} \\
\Longrightarrow &\neg\mathrm{C} \vee \neg\mathrm{O_2} \vee \mathrm{CO_2} \Longrightarrow \qquad \{\neg\mathrm{C}, \neg\mathrm{O_2}, \mathrm{CO_2}\}
\end{aligned}$$

(3)

$$\begin{aligned}
&\mathrm{CO_2} + \mathrm{H_2O} \to \mathrm{H_2CO_3} \\
\Longrightarrow &\mathrm{CO_2} \wedge \mathrm{H_2O} \to \mathrm{H_2CO_3} \\
\Longrightarrow &\neg\mathrm{CO_2} \vee \neg\mathrm{H_2O} \vee \mathrm{H_2CO_3} \Longrightarrow \quad \{\neg\mathrm{CO_2}, \neg\mathrm{H_2O}, \mathrm{H_2CO_3}\}
\end{aligned}$$

The elements availabe at the start are:

$$\begin{aligned} &\mathrm{MgO} \wedge \mathrm{H_2} \wedge \mathrm{O_2} \wedge \mathrm{C} \\ \Longrightarrow &\{\mathrm{MgO}\}, \{\mathrm{H_2}\}, \{\mathrm{O_2}\}\{\mathrm{C}\} \end{aligned}$$

We denote the above set of clauses with Σ. now we want to prove that $\Sigma \models \mathrm{H_2CO_3}$. The proof is constructed by showing that $\Sigma \cup \{\neg \mathrm{H_2CO_3}\}$ is unsatisfiable.

$$\begin{array}{ccc}
 & \{\neg \mathrm{H_2CO_3}\} & \{\neg \mathrm{CO_2}, \neg \mathrm{H_2O}, \mathrm{H_2CO_3}\} \\
\{\neg \mathrm{C}, \neg \mathrm{O_2}, \mathrm{CO2}\} & \{\neg \mathrm{CO_2}, \neg \mathrm{H_2O}\} & \\
 & \{\neg \mathrm{H_2O}, \neg \mathrm{C}, \neg \mathrm{O_2}\} & \{\mathrm{C}\} \\
\{\mathrm{O_2}\} & \{\neg \mathrm{H_2O}, \neg \mathrm{O_2}\} & \\
 & \{\neg \mathrm{H_2O}\} & \{\neg \mathrm{MgO}, \neg \mathrm{H_2}, H_2O\} \\
 & \{\neg \mathrm{MgO}, \neg \mathrm{H_2}\} & \{\mathrm{H_2}\} \\
 & \{\neg \mathrm{MgO}\} & \{\mathrm{MgO}\} \\
 & \square &
\end{array}$$