Tik-79.144 **Syksy 2000**
Logic in Computer Science: Foundations
Exercise 10

1. Examine if the given claim holds using semantic tableaux. If not, justify by giving a structure where the claim doesn't hold.

$$\{\exists x(P(x) \to Q(x)), \exists x(\neg R(x) \to \neg Q(X))\} \models \exists x(P(x) \to R(x))$$

2. A *directed* graph consists of a set of nodes connected by *directed* arcs. Assume that nodes are represented with constants $\{a, b, \ldots\}$ while arcs are represented with a binary predicate $A(x, y)$ = "there is an arc leading from the node x to the node y".

a) Define predicates $R_n(x, y)$ = "there is a directed route from the node x to the node y such that the route goes through n arcs", when n is $0, 1, 2, \ldots, k$. Describe the graph below using the predicate A.

$$a \;\; \begin{matrix} \longrightarrow \\ \longleftarrow \end{matrix} \;\; b \;\; \longrightarrow \;\; c$$

b) Use resolution to show that $\exists x(R_2(x, x) \wedge R_3(x, c))$ is a logical consequence of your description and the definitions of R_2 and R_3.