Tik-79.144 **Syksy 1999**
Logic in Computer Science: Foundations
Exercises 7
Solutions

1. We'll denote the cities with constant symbols h (Helsinki), l (London), r (Rovaniemi), n (New York), pa (Paris), pi (Petersburg) ja b (Berlin). The database for the given flights is:

$$\{L(h,l), L(h,r), L(h,n), L(n,pa), L(pi,b)\}$$

The given set of axioms is as follows:

$$\forall x \forall y (L(x,y) \rightarrow L(y,x))$$
$$\forall x \forall y (L(x,y) \rightarrow Y(x,y))$$
$$\forall x \forall y \forall z (Y(x,y) \wedge L(y,z) \rightarrow Y(x,z))$$

We use semantic tableaux to prove that it is possible to deduce that there is a connection from Rovaniemi to Paris (formalized: $Y(r,pa)$) given the database and the axioms.

The universal quantifiers are instantiated in one step in the tableaux. The "idea" of this proof is to show that:

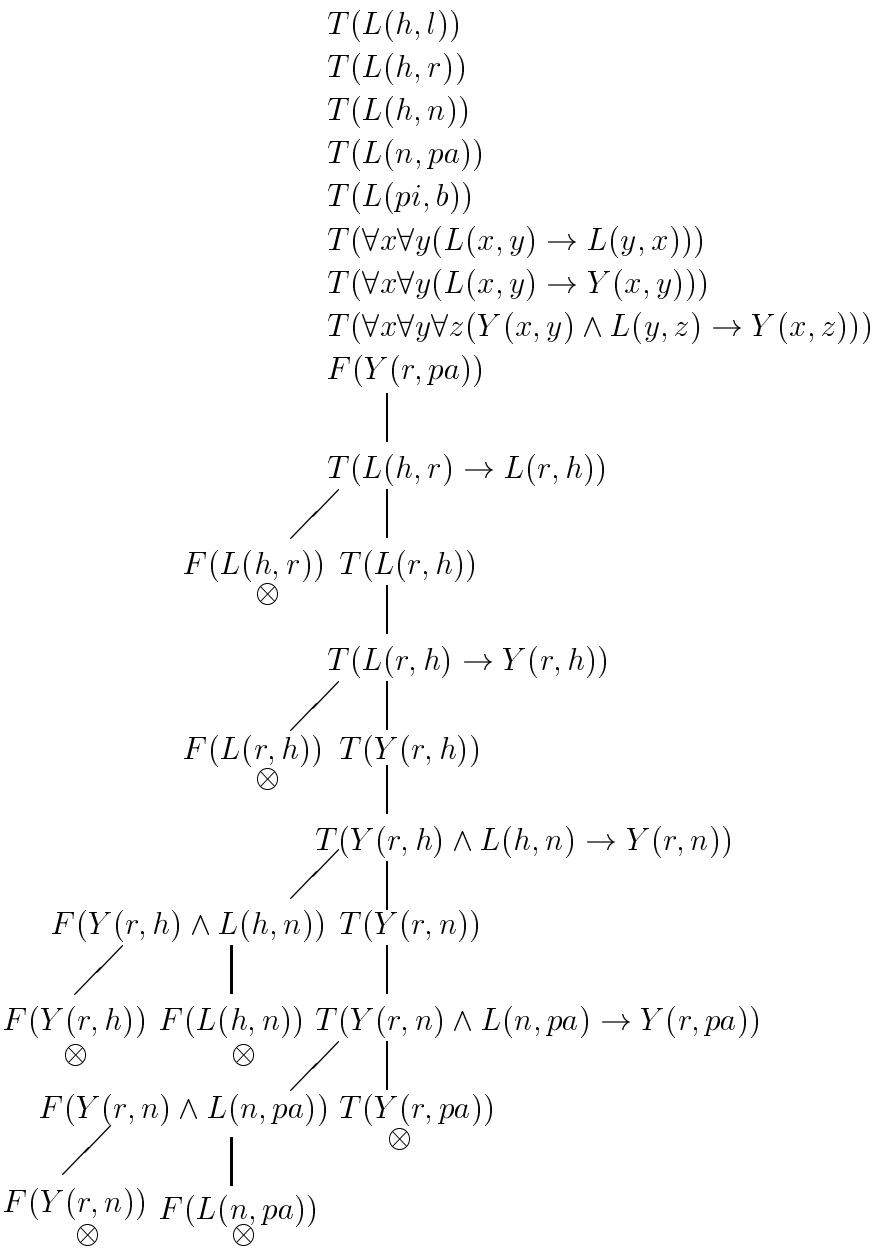
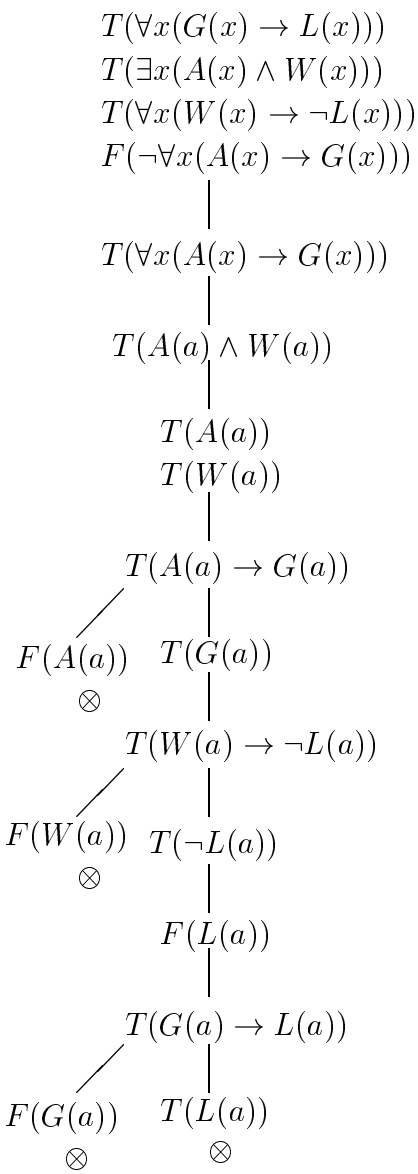
(i) There is a flight from Rovaniemi to Helsinki

(ii) There is a connection from Rovaniemi to Helsinki

(iii) There is a connection from Rovaniemi to New York

(iv) There is a connection from Rovaniemi to Paris

This strategy results in the smallest possible tableaux.


$T(L(h,l))$
$T(L(h,r))$
$T(L(h,n))$
$T(L(n,pa))$
$T(L(pi,b))$
$T(\forall x \forall y(L(x,y) \to L(y,x)))$
$T(\forall x \forall y(L(x,y) \to Y(x,y)))$
$T(\forall x \forall y \forall z(Y(x,y) \wedge L(y,z) \to Y(x,z)))$
$F(Y(r,pa))$
$T(L(h,r) \to L(r,h))$
$F(L(h,r))$
$T(L(r,h))$
$T(L(r,h) \to Y(r,h))$
$F(L(r,h))$
$T(Y(r,h))$
$T(Y(r,h) \wedge L(h,n) \to Y(r,n))$
$F(Y(r,h) \wedge L(h,n))$
$T(Y(r,n))$
$F(Y(r,h))$
$F(L(h,n))$
$T(Y(r,n) \wedge L(n,pa) \to Y(r,pa))$
$F(Y(r,n) \wedge L(n,pa))$
$T(Y(r,pa))$
$F(Y(r,n))$
$F(L(n,pa))$

2. Let predicates be $G(x)$ (x is guilty), $L(x)$ (x is liar), $A(x)$ (x is accused) ja $W(x)$ (x is a wittness). The premises are $\forall x(G(x) \rightarrow L(x))$, $\exists x(A(x) \wedge W(x))$ and $\forall x(W(x) \rightarrow \neg L(x))$. The desired consequence is $\neg\forall x(A(x) \rightarrow G(x))$. The semantic tableaux is: