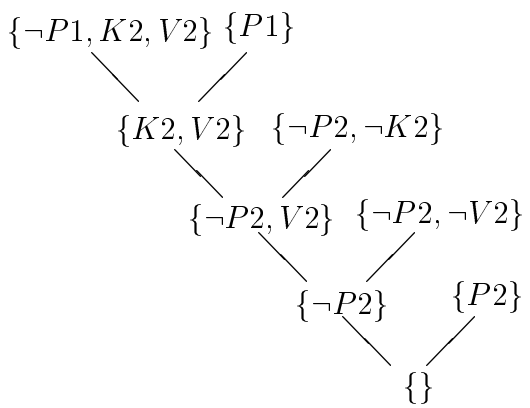
Tik-79.144 **Syksy 1999**
Logic in Computer Science: Foundations
Exercises 5
Solutions

1. Transform the propositions to conjunctive normal form and to a set of clauses. The last item of the table is the negation of the proposition to prove: $P1 \wedge P2$.

Proposition		Clauses
$Pi \vee Ki \vee Vi$		$\{Pi, Ki, Vi\}$
$Pi \rightarrow \neg Ki \wedge \neg Vi$	$\equiv \neg Pi \vee (\neg Ki \wedge \neg Vi)$	
	$\equiv (\neg Pi \vee \neg Ki) \wedge (\neg Pi \vee \neg Vi)$	$\{\neg Pi, \neg Ki\}, \{\neg Pi, \neg Vi\}$
$Ki \rightarrow \neg Pi \wedge \neg Vi$	$\equiv (\neg Ki \vee \neg Pi) \wedge (\neg Ki \vee \neg Vi)$	$\{\neg Pi, \neg Ki\}, \{\neg Ki, \neg Vi\}$
$Vi \rightarrow \neg Pi \wedge \neg Ki$	$\equiv (\neg Vi \vee \neg Pi) \wedge (\neg Vi \vee \neg Ki)$	$\{\neg Pi, \neg Vi\}, \{\neg Ki, \neg Vi\}$
$\neg(V1 \wedge V2)$	$\equiv \neg V1 \vee \neg V2$	$\{\neg V1, \neg V2\}$
$P1 \rightarrow (K2 \vee V2)$	$\equiv \neg P1 \vee K2 \vee V2$	$\{\neg P1, K2, V2\}$
$P2 \rightarrow (K1 \vee V1)$	$\equiv \neg P2 \vee K1 \vee V1$	$\{\neg P2, K1, V1\}$
$P1 \wedge P2$		$\{P1\}, \{P2\}$

The idea of the resolution proofs is to show that the above set of clauses is unsatisfiable, and so our supposition that it is possible to have both lights $P1$ and $P2$ on at the same time must be false. This is achieved by deriving the empty clause $\{\}$ from the set of clause.



2. The chemical reactions can be formalized as implications, which can then be transformed into clausul form. The resulting clauses are:

(1)

$$\begin{aligned}
&\mathrm{MgO} + \mathrm{H_2} \rightarrow \mathrm{Mg} + \mathrm{H_2O} \\
\Longrightarrow &\mathrm{MgO} \wedge \mathrm{H_2} \rightarrow \mathrm{Mg} \wedge \mathrm{H_2O} \\
\Longrightarrow &\neg\mathrm{MgO} \vee \neg\mathrm{H_2} \vee (\mathrm{Mg} \wedge \mathrm{H_2O}) \\
\Longrightarrow &(\neg\mathrm{MgO} \vee \neg\mathrm{H_2} \vee \mathrm{Mg}) \wedge (\neg\mathrm{MgO} \vee \neg\mathrm{H_2} \vee \mathrm{H_2O})
\end{aligned}$$

The first reaction results in two clauses: : $\{\neg\mathrm{MgO}, \neg\mathrm{H_2}, \mathrm{Mg}\}$ and $\{\neg\mathrm{MgO}, \neg\mathrm{H_2}, \mathrm{H_2O}\}$.

(2)

$$\begin{aligned}
&\mathrm{C} + \mathrm{O_2} \rightarrow \mathrm{CO_2} \\
\Longrightarrow &\mathrm{C} \wedge \mathrm{O_2} \rightarrow \mathrm{CO_2} \\
\Longrightarrow &\neg\mathrm{C} \vee \neg\mathrm{O_2} \vee \neg\mathrm{CO_2} \Longrightarrow \qquad \{\neg\mathrm{C}, \neg\mathrm{O_2}, \neg\mathrm{CO_2}\}
\end{aligned}$$

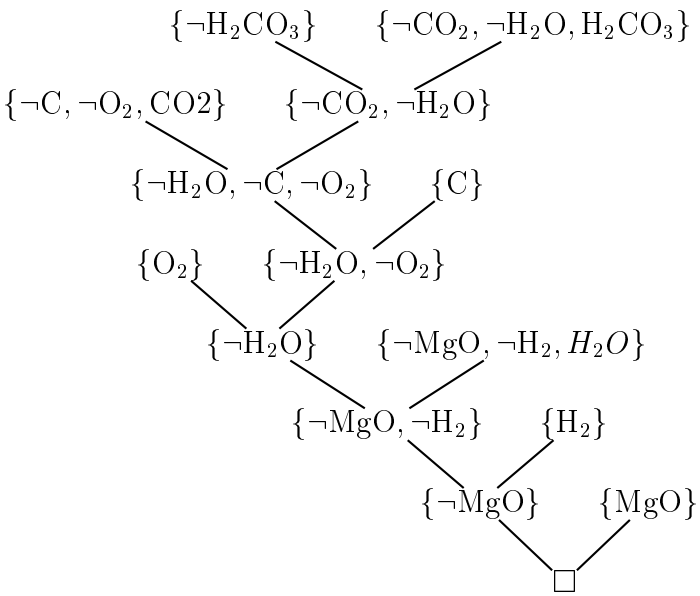
(3)

$$\begin{aligned}
&\mathrm{CO_2} + \mathrm{H_2O} \rightarrow \mathrm{H_2CO_3} \\
\Longrightarrow &\mathrm{CO_2} \wedge \mathrm{H_2O} \rightarrow \mathrm{H_2CO_3} \\
\Longrightarrow &\neg\mathrm{CO_2} \vee \neg\mathrm{H_2O} \vee \neg\mathrm{H_2CO_3} \Longrightarrow \quad \{\neg\mathrm{CO_2} \vee \neg\mathrm{H_2O} \vee \neg\mathrm{H_2CO_3}\}
\end{aligned}$$

The elements availabe at the start are:

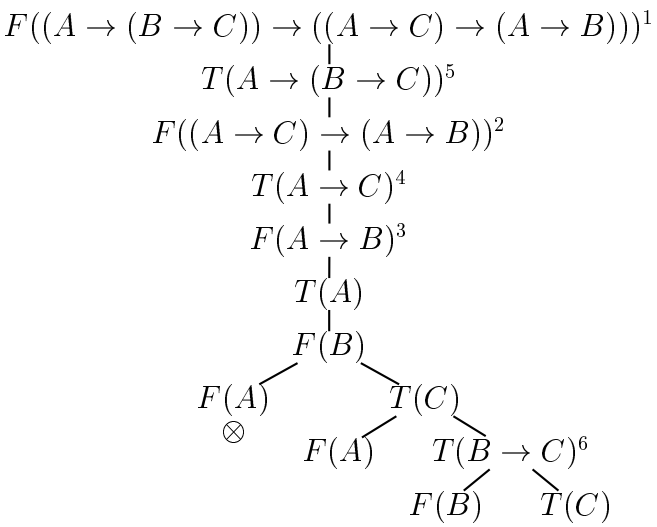
$$\begin{aligned}
&\mathrm{MgO} \wedge \mathrm{H_2} \wedge \mathrm{O_2} \wedge \mathrm{C} \\
\Longrightarrow &\{\mathrm{MgO}\}, \{\mathrm{H_2}\}, \{\mathrm{O_2}\}\{\mathrm{C}\}
\end{aligned}$$

We denote the above set of clauses with Σ. now we want to prove that $\Sigma \vDash \mathrm{H_2CO_3}$. The proof is constructed by showing that $\Sigma \cup \{\neg\mathrm{H_2CO_3}\}$ is unsatisfiable.



3. The notation $\vDash \phi$ means that ϕ is valid. So in both cases the claim is that the propositions are valid. We use semantic tableaux to examine the validity of the propositions in normal way:

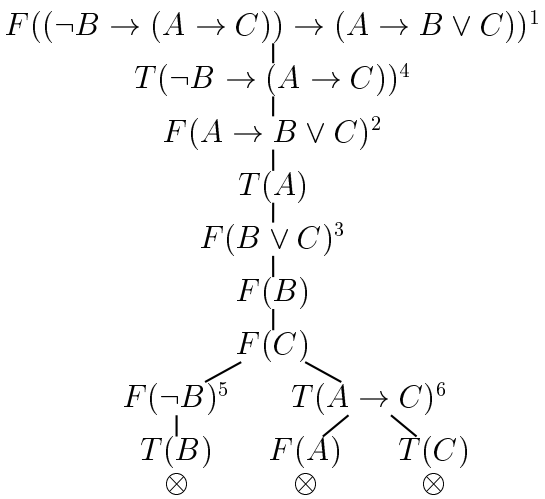
a)



Because the tableau is not contradictionary, the proposition is not valid. The counter example can be read from an open branch of the tableau. If for some atomic symbol P there is $T(P)$ in the branch we add $\mathcal{V}(P) = T$ to the counter example, and if there is $F(P)$, we add $\mathcal{V}(P) = F$. From the rightmost branch we get:

$$\mathcal{V}(A) = \mathcal{V}(C) = T, \mathcal{V}(B) = F$$

b)



As all branches are contradictionary the proposition is valid, and the claim is true.