Tik-79.144 **Syksy 1999**
Logic in Computer Science: Foundations
Exercise 5
12. ja 15.10.1999

1. A few weeks ago a traffic light system was modeled with following propositions:

$P1 \vee K1 \vee V1$	$P2 \vee K2 \vee V2$	$\neg(V1 \wedge V2)$
$P1 \rightarrow \neg K1 \wedge \neg V1$	$P2 \rightarrow \neg K2 \wedge \neg V2$	$P1 \rightarrow (K2 \vee V2)$
$K1 \rightarrow \neg P1 \wedge \neg V1$	$K2 \rightarrow \neg P2 \wedge \neg V2$	$P2 \rightarrow (K1 \vee V1)$
$V1 \rightarrow \neg P1 \wedge \neg K1$	$V2 \rightarrow \neg P2 \wedge \neg K2$	

The atomic propositions P_i, K_i and V_i mean that the light post i has (in following order) a red, a yellow and a green light on. Transform the propositions into clausuls and prove with resolution that both red lights are not on at the same time.

2. One successful application of expert systems has been analyzing the problem of which chemical syntheses are possible. Consider the following chemical reactions:

(1) $\mathrm{MgO} + \mathrm{H_2} \rightarrow \mathrm{Mg} + \mathrm{H_2O}$

(2) $\mathrm{C} + \mathrm{O_2} \rightarrow \mathrm{CO_2}$

(3) $\mathrm{CO_2} + \mathrm{H_2O} \rightarrow \mathrm{H_2CO_3}$

a) Represent these rules and the assumptions that we have some $\mathrm{MgO}, \mathrm{H_2}, \mathrm{O_2}$ and C by propositional logic formulas.

b) Give a resolution proof that we can get some $\mathrm{H_2CO_3}$.

3. Examine if the given claims hold using semantic tableaux. If not, justify by giving a valuation where the claim doesn't hold (a counter example)

a) $\models (A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow C) \rightarrow (A \rightarrow B))$

b) $\models (\neg B \rightarrow (A \rightarrow C)) \rightarrow (A \rightarrow (B \vee C))$