Tik-79.144 **Syksy 1999**
Logic in Computer Science: Foundations
Exercise 3
29.9. ja 1.10.1999

1. Use semantic tableaux to prove following propositions

a) $A \to (B \to B)$,

b) $(A \to B) \wedge (B \to C) \to (A \to C)$,

c) $(A \to B) \wedge (A \to C) \to (A \to B \wedge C)$ ja

d) $(A \to C) \wedge (B \to C) \wedge (A \vee B) \to C$.

2. Use a semantic tableaux to check whether following claims hold. If not, give a counterexample.

a) $\{B \to A, C \to B, (C \to A) \to D\} \models D$

b) $\{A \to C, A \vee B, \neg D \to \neg B\} \models C \to D$

3. We can prove all valid statements of propositional logic using following axioms:

A1: $P \to (Q \to P)$,
A2: $(P \to (Q \to R)) \to ((P \to Q) \to (P \to R))$ ja
A3: $(\neg Q \to \neg P) \to ((\neg Q \to P) \to Q)$.

and *modus ponens* -rule of inference:

$$\frac{P \quad P \to Q}{Q}$$

Use the axiom schema to prove following propositions:

a) $\vdash P \to P$,

b) $\{P \to Q, Q \to R\} \vdash P \to R$,

c) $\{P, Q \to (P \to R)\} \vdash Q \to R$ ja

d) $\vdash \neg\neg P \to P$.