Tik-79.144 **Syksy 1999**
Logic in Computer Science: Foundations
Exercise 11

1. Use *otter* program to prove the following claim.

$$\{\forall x(P(x) \to Q(x)), \forall x(\neg R(x) \to \neg Q(X))\} \models \forall x(P(x) \to R(x))$$

2. We are given following sentences[1]:

1. $\forall x(\text{Close}(\text{m}, x) \to \text{Reach}(\text{m}, x))$,
2. $\forall x(\text{Tall}(x) \wedge \text{On}(\text{m}, x) \wedge \text{Under}(x, \text{b}) \to \text{Close}(\text{m}, \text{b}))$,
3. $\text{Tall}(\text{c})$,
4. $\text{Climb}(\text{m}, \text{c})$,
5. $\forall z\text{Moves}(\text{m}, \text{c}, z)$,
6. $\forall x(\text{Climb}(\text{m}, x) \to \text{On}(\text{m}, x))$,
7. $\forall x \forall y \forall z(\text{Moves}(x, y, z) \to \text{Close}(y, z) \vee \text{Under}(y, z))$ ja
8. $\neg\text{Close}(\text{c}, \text{b})$.

The meaning of the predicates and the constants is following:

1. $\text{Tall}(x)$ = "x is tall",
2. $\text{Climb}(x, y)$ = "x can climb onto y",
3. $\text{Close}(x, y)$ = "x is near y",
4. $\text{On}(x, y)$ = "x is on y",
5. $\text{Reach}(x, y)$ = "x reaches y",
6. $\text{Under}(x, y)$ = "x is under y",
7. $\text{Moves}(x, y, z)$ = "x (can) move y near z",
8. m = "monkey",
9. c = "chair" and
10. b = "banana".

Think what is the meaning of the sentences. Prove with an `otter`-program, that $\exists x\text{Reach}(m, x)$ is a logical consequence of the set of sentences.

[1] Esimerkki on kirjasta R.D. Dowsing, V.J. Rayward-Smith, C.D. Walter: A First Course in Formal Logic and its Applications in Computer Science