Tik-79.144 **Syksy 1998**
Logic in Computer Science: Foundations
Exercises 7
Solutions

1. We'll denote the cities with constant symbols h (Helsinki), l (London), r (Rovaniemi), n (New York), pa (Paris), pi (Petersburg) ja b (Berlin). The database for the given flights is:

$$\{L(h,l), L(h,r), L(h,n), L(n,pa), L(pi,b)\}$$

The given set of axioms is as follows:

$$\begin{array}{l} \forall x \forall y (L(x,y) \to L(y,x)) \\ \forall x \forall y (L(x,y) \to Y(x,y)) \\ \forall x \forall y \forall z (Y(x,y) \land L(y,z) \to Y(x,z)) \end{array}$$

We use semantic tableaux to prove that it is possible to deduce that there is a connection from Rovaniemi to Paris (formalized: $Y(r,pa)$) given the database and the axioms.

The universal quantifiers are instantiated in one step in the tableaux. The "idea" of this proof is to show that:

(i) There is a flight from Rovaniemi to Helsinki

(ii) There is a connection from Rovaniemi to Helsinki

(iii) There is a connection from Rovaniemi to New York

(iv) There is a connection from Rovaniemi to Paris

This strategy results in the smallest possible tableaux.

$$T(L(h,l))$$
$$T(L(h,r))$$
$$T(L(h,n))$$
$$T(L(n,pa))$$
$$T(L(pi,b))$$
$$T(\forall x \forall y(L(x,y) \rightarrow L(y,x)))$$
$$T(\forall x \forall y(L(x,y) \rightarrow Y(x,y)))$$
$$T(\forall x \forall y \forall z(Y(x,y) \wedge L(y,z) \rightarrow Y(x,z)))$$
$$F(Y(r,pa))$$

$$T(L(h,r) \rightarrow L(r,h))$$

$$\underset{\otimes}{F(L(h,r))} \quad T(L(r,h))$$

$$T(L(r,h) \rightarrow Y(r,h))$$

$$\underset{\otimes}{F(L(r,h))} \quad T(Y(r,h))$$

$$T(Y(r,h) \wedge L(h,n) \rightarrow Y(r,n))$$

$$F(Y(r,h) \wedge L(h,n)) \quad T(Y(r,n))$$

$$\underset{\otimes}{F(Y(r,h))} \quad \underset{\otimes}{F(L(h,n))} \quad T(Y(r,n) \wedge L(n,pa) \rightarrow Y(r,pa))$$

$$F(Y(r,n) \wedge L(n,pa)) \quad \underset{\otimes}{T(Y(r,pa))}$$

$$\underset{\otimes}{F(Y(r,n))} \quad \underset{\otimes}{F(L(n,pa))}$$

2. The proof is as follows (constants $c_1, \ldots, c_6$ are new constants):

$$F(\forall x \exists y \neg \forall z P(x,y,z) \leftrightarrow \forall x \exists y \exists z \neg P(x,y,z))$$

$$\begin{array}{cc}
T(\forall x \exists y \neg \forall z P(x,y,z)) & F(\forall x \exists y \neg \forall z P(x,y,z)) \\
F(\forall x \exists y \exists z \neg P(x,y,z)) & T(\forall x \exists y \exists z \neg P(x,y,z)) \\
| & | \\
F(\exists y \exists z \neg P(c_1,y,z)) & F(\exists y \neg \forall z P(c_4,y,z)) \\
| & | \\
T(\exists y \neg \forall z P(c_1,y,z)) & T(\exists y \exists z \neg P(c_4,y,z)) \\
| & | \\
T(\neg \forall z P(c_1,c_2,z)) & T(\exists z \neg P(c_4,c_5,z)) \\
| & | \\
F(\exists z \neg P(c_1,c_2,z)) & F(\neg \forall z P(c_4,c_5,z)) \\
| & | \\
F(\forall z P(c_1,c_2,z)) & T(\neg P(c_4,c_5,c_6)) \\
| & | \\
F(P(c_1,c_2,c_3)) & T(\forall z P(c_4,c_5,z)) \\
| & | \\
F(\neg P(c_1,c_2,c_3)) & F(P(c_4,c_5,c_6)) \\
| & | \\
T(P(c_1,c_2,c_3)) & T(P(c_4,c_5,c_6)) \\
\otimes & \otimes
\end{array}$$

3. Let predicates be $G(x)$ (x is guilty), $L(x)$ (x is liar), $A(x)$ (x is accused) ja $W(x)$ (x is a wittness). The premises are $\forall x(G(x) \to L(x))$, $\exists x(A(x) \land W(x))$ and $\forall x(W(x) \to \neg L(x))$. The desired consequence is $\neg\forall x(A(x) \to G(x))$. The semantic tableaux is:

$$
\begin{array}{c}
T(\forall x(G(x) \to L(x))) \\
T(\exists x(A(x) \land W(x))) \\
T(\forall x(W(x) \to \neg L(x))) \\
F(\neg\forall x(A(x) \to G(x))) \\
| \\
T(\forall x(A(x) \to G(x))) \\
| \\
T(A(a) \land W(a)) \\
| \\
T(A(a)) \\
T(W(a)) \\
| \\
T(A(a) \to G(a))
\end{array}
$$

$$
\begin{array}{cc}
/ & | \\
F(A(a)) & T(G(a)) \\
\otimes & | \\
 & T(W(a) \to \neg L(a))
\end{array}
$$

$$
\begin{array}{cc}
/ & | \\
F(W(a)) & T(\neg L(a)) \\
\otimes & | \\
 & F(L(a)) \\
 & | \\
 & T(G(a) \to L(a))
\end{array}
$$

$$
\begin{array}{cc}
/ & | \\
F(G(a)) & T(L(a)) \\
\otimes & \otimes
\end{array}
$$