Tik-79.144 **Syksy 1998**
Logic in Computer Science: Foundations
Exercises 5
Solutions

1. The notation $\models \phi$ means that ϕ is valid. So in both cases the claim is that the propositions are valid. We use semantic tableaux to examine the validity of the propositions in normal way:

a) $F((A \to (B \to C)) \to ((A \to C) \to (A \to B)))^1$

$$T(A \to (B \to C))^5$$
$$F((A \to C) \to (A \to B))^2$$
$$T(A \to C)^4$$
$$F(A \to B)^3$$
$$T(A)$$
$$F(B)$$
$$F(A) \qquad\qquad T(C)$$
$$\otimes \qquad\qquad F(A) \qquad T(B \to C)^6$$
$$F(B) \qquad T(C)$$

Because the tableau is not contradictionary, the proposition is not valid. The counter example can be read from an open branch of the tableau. If for some atomic symbol P there is $T(P)$ in the branch we add $\mathcal{V}(P) = T$ to the counter example, and if there is $F(P)$, we add $\mathcal{V}(P) = F$. From the rightmost branch we get:

$$\mathcal{V}(A) = \mathcal{V}(C) = T, \mathcal{V}(B) = F$$

b) $F((\neg B \to (A \to C)) \to (A \to B \vee C))^1$

$$T(\neg B \to (A \to C))^4$$

$$F(A \to B \vee C)^2$$

$$T(A)$$

$$F(B \vee C)^3$$

$$F(B)$$

$$F(C)$$

$$\begin{array}{ccc} F(\neg B)^5 & \multicolumn{2}{c}{T(A \to C)^6} \\ T(B) & F(A) & T(C) \\ \otimes & \otimes & \otimes \end{array}$$

As all branches are contradictionary the proposition is valid, and the claim is true.

2. The chemical reactions can be formalized as implications, which can then be transformed into clausul form. The resulting clausuls are:

(1)

$$\begin{aligned} & \mathrm{MgO} + \mathrm{H_2} \to \mathrm{Mg} + \mathrm{H_2O} \\ \Longrightarrow & \mathrm{MgO} \wedge \mathrm{H_2} \to \mathrm{Mg} \wedge \mathrm{H_2O} \\ \Longrightarrow & \neg\mathrm{MgO} \vee \neg\mathrm{H_2} \vee (\mathrm{Mg} \wedge \mathrm{H_2O}) \\ \Longrightarrow & (\neg\mathrm{MgO} \vee \neg\mathrm{H_2} \vee \mathrm{Mg}) \wedge (\neg\mathrm{MgO} \vee \neg\mathrm{H_2} \vee \mathrm{H_2O}) \end{aligned}$$

The first reaction results in two clausuls: : $\{\neg\mathrm{MgO}, \neg\mathrm{H_2}, \mathrm{Mg}\}$ and $\{\neg\mathrm{MgO}, \neg\mathrm{H_2}, \mathrm{H_2O}\}$.

(2)

$$\begin{aligned} & \mathrm{C} + \mathrm{O_2} \to \mathrm{CO_2} \\ \Longrightarrow & \mathrm{C} \wedge \mathrm{O_2} \to \mathrm{CO_2} \\ \Longrightarrow & \neg\mathrm{C} \vee \neg\mathrm{O_2} \vee \neg\mathrm{CO_2} \Longrightarrow \qquad \{\neg\mathrm{C}, \neg\mathrm{O_2}, \neg\mathrm{CO_2}\} \end{aligned}$$

(3)

$$\begin{aligned} & \mathrm{CO_2} + \mathrm{H_2O} \to \mathrm{H_2CO_3} \\ \Longrightarrow & \mathrm{CO_2} \wedge \mathrm{H_2O} \to \mathrm{H_2CO_3} \\ \Longrightarrow & \neg\mathrm{CO_2} \vee \neg\mathrm{H_2O} \vee \neg\mathrm{H_2CO_3} \Longrightarrow \quad \{\neg\mathrm{CO_2} \vee \neg\mathrm{H_2O} \vee \neg\mathrm{H_2CO_3}\} \end{aligned}$$

The elements availabe at the start are:

$$\begin{aligned} &\mathrm{MgO} \wedge \mathrm{H_2} \wedge \mathrm{O_2} \wedge \mathrm{C} \\ \Longrightarrow &\{\mathrm{MgO}\}, \{\mathrm{H_2}\}, \{\mathrm{O_2}\}\{\mathrm{C}\} \end{aligned}$$

We denote the above set of clausuls with Σ. now we want to prove that $\Sigma \models \mathrm{H_2CO_3}$. The proof is constructed by showing that $\Sigma \cup \{\neg \mathrm{H_2CO_3}\}$ is unsatisfiable.