Tik-79.144 **Syksy 1998**
Logic in Computer Science: Foundations
Exercises 4
Solutions

1. a)

1.	$(P \to ((P \to P) \to P))$	[A1]
2.	$((P \to ((P \to P) \to P)) \to$ $((P \to (P \to P)) \to (P \to P)))$	[A2]
3.	$((P \to (P \to P)) \to (P \to P))$	[MP:1,2]
4.	$(P \to (P \to P))$	[A1]
5.	$(P \to P)$	[MP:3,4]

b)

1.	$(Q \to R)$	[P2]
2.	$((Q \to R) \to (P \to (Q \to R)))$	[A1]
3.	$(P \to (Q \to R))$	[MP:1,2]
4.	$((P \to (Q \to R)) \to ((P \to Q) \to (P \to R)))$	[A2]
5.	$((P \to Q) \to (P \to R))$	[MP:3,4]
6.	$(P \to Q)$	[P1]
7.	$(P \to R)$	[MP:5,6]

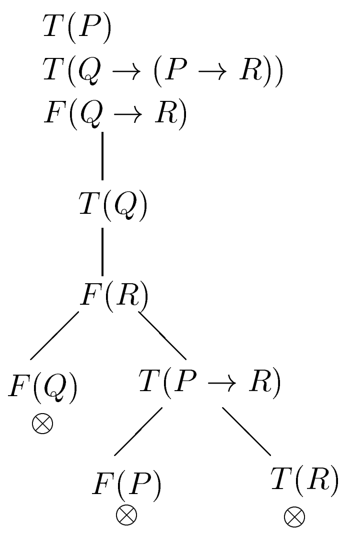
c)

1.	P	[P1]
2.	$(Q \to (P \to R))$	[P2]
3.	$(P \to (Q \to P))$	[A1]
4.	$(Q \to P)$	[MP:1,3]
5.	$((Q \to (P \to R)) \to ((Q \to P) \to (Q \to R)))$	[A2]
6.	$((Q \to P) \to (Q \to R))$	[MP:2,5]
7.	$(Q \to R)$	[MP:4,6]

d) We may use the above results when proving the last proposition.

1.	$((\neg P \to \neg\neg P) \to ((\neg P \to \neg P) \to P))$	[A3]
2.	$(\neg P \to \neg P)$	[(a)]
3.	$((\neg P \to \neg\neg P) \to P)$	[(c):1,2]
4.	$(\neg\neg P \to (\neg P \to \neg\neg P))$	[A1]
5.	$(\neg\neg P \to P)$	[(b):4,3]

2. The semantic tableau is constructed in normal way. The claim $\{P, Q \rightarrow (P \rightarrow R)\} \vdash Q \rightarrow R$ is true, because all branches of the tableaux are contradictionary.

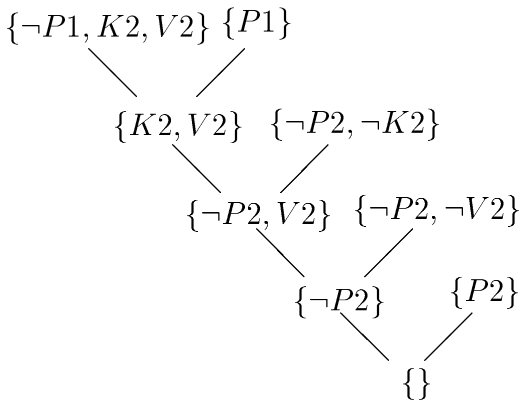


The proofs are much more straightforward when done with semantic tableaux, compared with using the axiom system. When using axioms, it is not at all clear which axiom has to be used at each step.

3. Transform the propositions to conjunctive normal form and to a set of clausuls. The last item of the table is the negation of the proposition to prove: $P1 \wedge P2$.

Proposition		Clausuls
$Pi \vee Ki \vee Vi$		$\{Pi, Ki, Vi\}$
$Pi \rightarrow \neg Ki \wedge \neg Vi$	$\equiv \neg Pi \vee (\neg Ki \wedge \neg Vi)$ $\equiv (\neg Pi \vee \neg Ki) \wedge (\neg Pi \vee \neg Vi)$	$\{\neg Pi, \neg Ki\}, \{\neg Pi, \neg Vi\}$
$Ki \rightarrow \neg Pi \wedge \neg Vi$	$\equiv (\neg Ki \vee \neg Pi) \wedge (\neg Ki \vee \neg Vi)$	$\{\neg Pi, \neg Ki\}, \{\neg Ki, \neg Vi\}$
$Vi \rightarrow \neg Pi \wedge \neg Ki$	$\equiv (\neg Vi \vee \neg Pi) \wedge (\neg Vi \vee \neg Ki)$	$\{\neg Pi, \neg Vi\}, \{\neg Ki, \neg Vi\}$
$\neg(V1 \wedge V2)$	$\equiv \neg V1 \vee \neg V2$	$\{\neg V1, \neg V2\}$
$P1 \rightarrow (K2 \vee V2)$	$\equiv \neg P1 \vee K2 \vee V2$	$\{\neg P1, K2, V2\}$
$P2 \rightarrow (K1 \vee V1)$	$\equiv \neg P2 \vee K1 \vee V1$	$\{\neg P2, K1, V1\}$
$P1 \wedge P2$		$\{P1\}, \{P2\}$

The idea of the resolution proofs is to show that the above set of clausuls is unsatisfiable, and so our supposition that it is possible to have both lights $P1$ and $P2$ on at the same time must be false. This is achieved by deriving the empty clausul $\{\}$ from the set of clausuls.


{¬P1, K2, V2}
{P1}
{K2, V2}
{¬P2, ¬K2}
{¬P2, V2}
{¬P2, ¬V2}
{¬P2}
{P2}
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