Tik-79.144 **Syksy 1998**
Logic in Computer Science: Foundations
Exercises 1
Solutions

1. a) $\neg A \rightarrow \neg B$, when
$A =$ "You help"
$B =$ "I can finish my work"

b) $A \vee B \vee C$, when
$A =$ "I walk to work"
$B =$ "I ride a bicycle to work"
$C =$ "Sometimes I drive a car to work"

c) Either: A, when
$A =$ "Merja and Arto are coming to visit us"
or: $A \wedge B$, when
$A =$ "Merja is coming to visit us"
$B =$ "Arto is coming to visit us"

d) For example: $A \rightarrow \neg B$ or $A \wedge \neg B$, when
$A =$ "You have been naughty"
$B =$ "You will get dessert"

e) For example: $A \wedge B$, when
$A =$ "The manual was long"
$B =$ "The manual was not long enough"

f) $A \vee \neg A \rightarrow \neg B \vee (C \wedge D)$, when
$A =$ "Somebody asks me"
$B =$ "He should buy a car"
$C =$ "He should live far from his workplace"
$D =$ "The price of benzine should get cheaper"

2. Prove the claim using induction on the length of proposition

Basic case: A proposition of the length 1 is a propositional symbol and it contains 0 parentheses (0 is an even number).

Induction hypothesis: If the length of the proposition is less than n, then it contains an even number of parenthesis.

Induction step: Let the length of the proposition to be n ($n > 1$). Then the proposition is of the form $(\neg\alpha)$, $(\alpha \vee \beta)$, $(\alpha \wedge \beta)$, $(\alpha \rightarrow \beta)$ or $(\alpha \leftrightarrow \beta)$. Here α and β are some propositions that are shorter than n. By induction hypothesis they contain an even number of parenthesis and so the proposition itself contains also an even number of parenthesis. This completes the proof by induction.

3. Using the common definitions for connective precedence:

a) $A \to (B \wedge C) \vee D$

b) $(A \to B) \wedge (B \to C) \to (A \to C)$

c) $(A \wedge (B \vee C)) \vee (A \wedge (C \vee D))$

d) $\neg(A \wedge B) \leftrightarrow (B \to C) \wedge A$

e) $\neg A \wedge \neg B \to \neg(A \vee B)$