Tik-79.144 **Syksy 1998**
Logic in Computer Science: Foundations
Exercise 4
6. ja 8.10.1998

1. We can prove all valid statements of propositional logic using following axioms:

$$\begin{aligned}
&\text{A1:} \quad P \to (Q \to P), \\
&\text{A2:} \quad (P \to (Q \to R)) \to ((P \to Q) \to (P \to R)) \text{ ja} \\
&\text{A3:} \quad (\neg Q \to \neg P) \to ((\neg Q \to P) \to Q).
\end{aligned}$$

and *modus ponens* -rule of inference:

$$\frac{P \quad P \to Q}{Q}$$

Use the axiom schema to prove following propositions:

a) $\vdash P \to P$,

b) $\{P \to Q, Q \to R\} \vdash P \to R$,

c) $\{P, Q \to (P \to R)\} \vdash Q \to R$ ja

d) $\vdash \neg\neg P \to P$.

2. Prove the (c)-part of the above exercise using semantic tableaux. What are the strong points of the semantic tableaux compared with axiom schemas?

3. A few weeks ago a traffic light system was modeled with following propositions:

$P1 \lor K1 \lor V1$	$P2 \lor K2 \lor V2$	$\neg(V1 \land V2)$
$P1 \to \neg K1 \land \neg V1$	$P2 \to \neg K2 \land \neg V2$	$P1 \to (K2 \lor V2)$
$K1 \to \neg P1 \land \neg V1$	$K2 \to \neg P2 \land \neg V2$	$P2 \to (K1 \lor V1)$
$V1 \to \neg P1 \land \neg K1$	$V2 \to \neg P2 \land \neg K2$	

The atomic propositions P_i, K_i and V_i mean that the light post i has (in following order) a red, a yellow and a green light on. Transform the propositions into clausuls and prove with resolution that both red lights are not on at the same time.