Tik-79.144 **Syksy 1998**
Logic in Computer Science: Foundations
Exercise 2
22. ja 24.9.1998

1. An engineer designed a specification for two traffic light posts positioned in the intersection of two one-way streets:

(i) Both lights have a green, a yellow and a red light. Exactly one of the lights is lit on both posts all times.

(ii) Both green lights are not on at the same time.

(iii) If one lamp post has the red light on, then the other has either green or yellow light on.

a) Formalize the above requirements as a set of propositional logic statements.

b) Give a model for the set of statements such that the proposition "the yellow light is on on both traffic lights" is true.

c) Give a valuation such that the set of statements is not satisfied.

d) Are the requirements complete enough for a real life situation?

2. Prove that the proposition "both red lights are not on at the same time" follows logically from the above set of statements by using (a) truth tables and (b) semantic tableaux.

3. Peirce's arrow is defined as follows:

$$(A \downarrow B) \Leftrightarrow_{def} \neg A \wedge \neg B.$$

Show that propositional connectives $\neg$, $\wedge$ ja $\vee$ can all be defined with Peirce's arrow.

Also give semantic tableau rules for the Peirce's arrow.