Tik-79.144 **Syksy 1998**
Logic in Computer Science: Foundations
Exercise 10

1. A *directed* graph consists of a set of nodes connected by *directed* arcs. Assume that nodes are represented with constants $\{a, b, \ldots\}$ while arcs are represented with a binary predicate $A(x, y) =$ "there is an arc leading from the node x to the node y".

a) Define predicates $R_n(x, y) =$ "there is a directed route from the node x to the node y such that the route goes through n arcs", when n is $0, 1, 2, \ldots, k$. Describe the graph below using the predicate A.

$$a \;\; \underset{\longleftarrow}{\overset{\longrightarrow}{}} \;\; b \;\; \longrightarrow \;\; c$$

b) Use resolution to show that $\exists x(R_2(x, x) \wedge R_3(x, c))$ is a logical consequence of your description and the definitions of R_2 and R_3.

2. We are given following sentences[1]:

1. $\forall x(\mathrm{Close}(\mathrm{m}, x) \to \mathrm{Reach}(\mathrm{m}, x))$,
2. $\forall x(\mathrm{Tall}(x) \wedge \mathrm{On}(\mathrm{m}, x) \wedge \mathrm{Under}(x, \mathrm{b}) \to \mathrm{Close}(\mathrm{m}, \mathrm{b}))$,
3. $\mathrm{Tall}(\mathrm{c})$,
4. $\mathrm{Climb}(\mathrm{m}, \mathrm{c})$,
5. $\forall z \mathrm{Moves}(\mathrm{m}, \mathrm{c}, z)$,
6. $\forall x(\mathrm{Climb}(\mathrm{m}, x) \to \mathrm{On}(\mathrm{m}, x))$,
7. $\forall x \forall y \forall z(\mathrm{Moves}(x, y, z) \to \mathrm{Close}(y, z) \vee \mathrm{Under}(y, z))$ ja
8. $\neg \mathrm{Close}(\mathrm{c}, \mathrm{b})$.

The meaning of the predicates and the constants is following:

1. $\mathrm{Tall}(x) =$ "x is tall",
2. $\mathrm{Climb}(x, y) =$ "x can climb onto y",
3. $\mathrm{Close}(x, y) =$ "x is near y",
4. $\mathrm{On}(x, y) =$ "x is on y",
5. $\mathrm{Reach}(x, y) =$ "x reaches y",
6. $\mathrm{Under}(x, y) =$ "x is under y",
7. $\mathrm{Moves}(x, y, z) =$ "x (can) move y near z",
8. $\mathrm{m} =$ "monkey",
9. $\mathrm{c} =$ "chair" and
10. $\mathrm{b} =$ "banana".

Think what is the meaning of the sentences. Prove with an `otter`-program, that $\exists x \mathrm{Reach}(m, x)$ is a logical consequence of the set of sentences.

[1] Esimerkki on kirjasta R.D. Dowsing, V.J. Rayward-Smith, C.D. Walter: A First Course in Formal Logic and its Applications in Computer Science