Helsinki University of Technology
Laboratory for Theoretical Computer Science
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T-79.1002 Introduction to Theoretical Computer Science Y (2 cr)
Exam Tue 6 March 2007 9 a.m. to noon

Write on every answer sheet:
- Name, degree programme, student number
- The text: "T-79.1002 Introduction to Theoretical Computer Science Y 6.3.2007"
- The total number of answer sheets submitted for grading

1. Describe the following languages in terms of deterministic finite automata:

   (a) $A = \{w \in \{a,b\}^* \mid w \text{ contains an odd number of } a\text{'s}\}$; *3p.*

   (b) $L = \{w \in \{0,1\}^* \mid w \text{ contains } 010 \text{ as a substring}\}$; and *3p.*

   (c) $\overline{L} = \{w \in \{0,1\}^* \mid w \text{ does not contain } 010 \text{ as a substring}\}$. *4p.*

   Hint: It may be easiest to derive the solution to (c) from the solution to (b).

2. Describe the following languages in terms of regular expressions:

   (a) $A = \{w \in \{a,b\}^* \mid w \text{ contains an odd number of } a\text{'s}\}$; *3p.*

   (b) $L = \{w \in \{0,1\}^* \mid w \text{ contains } 010 \text{ as a substring}\}$; and *3p.*

   (c) $\overline{L} = \{w \in \{0,1\}^* \mid w \text{ does not contain } 010 \text{ as a substring}\}$. *4p.*

   Hint: It may be easiest to derive the solution to (2c) from the solution to (1c).

3. Consider the *properly nested* strings of parentheses and angle brackets. For example, `([])[]` and `[([])]` are strings of properly nested parenthesis, but `([`, `[)`, and `])([` are not. More formally, the properly nested strings can be defined inductively: ε is a string of properly nested parenthesis, and if x and y are strings of properly nested parenthesis, then so are (x), $[y]$, and xy.

   (a) Design a context-free grammar that produces L. *5 p.*

   (b) Give the parse trees of the strings `([])[]` and `[([])]` in your grammar. *5 p.*

4. Closure properties of language classes: Given a language L over an alphabet Σ, let $L^R = \{w^R \mid w \in L\}$ be the language obtained by reversing each string in L. Here w^R is the reverse of w (for example, $(gnat)^R = tang$).

   (a) Show that if L is regular, then L^R is regular. *5 p.*

   (b) Show that if L is context-free, then L^R is context-free. *5 p.*

Total 40p.