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T-79.1002 Introduction to Theoretical Computer Science Y (2 ECTS)
Exam Thu 19 May 2006, 3–6 p.m.

Write down on each answer sheet:
- Your name, department, and student id
- The text: "T-79.1002 Introduction to Theoretical Computer Science Y 19.5.2006"
- The total number of answer sheets you are submitting for grading

Note that you CANNOT use this exam to compensate for course T-79.148 in the pre-2005 study requirements!!! If you want to take an exam for this course, or the post-2005 two-period course T-79.1001, please ask for another exam sheet!!!

1. Which of the following claims are true (T) and which false (F):
   (a) Any language recognised (decided) by a nondeterministic finite automaton can be described by a regular expression. *2p.*
   (b) The complement of any context-free language is regular. *2p.*
   (c) The intersection of any two regular languages can be recognised (decided) by a nondeterministic finite automaton. *2p.*
   (d) The union of any two regular languages is context-free. *2p.*

2. Show that each of the following languages is regular, by describing it either in terms of a regular expression or in terms of a finite automaton:
   (a) $\{w \in \{0,1\}^* \mid w$ contains three consequent zeros or three consequent ones (or both)$\}$; *4p.*
   (b) $\{w \in \{0,1\}^* \mid w$ contains neither three consequent zeros nor three consequent ones$\}$; *4p.*
   (c) $\{w \in \{0,1\}^* \mid$ the number of ones in w is a multiple of three (possibly zero)$\}$; *4p.*
   (d) $\{w \in \{0,1\}^* \mid |w| \geq 3$ and the third-to-last symbol in w is a $1\}$. *4p.*

3. (a) Show that the following context-free grammar is ambiguous:

$$S \to aSb \mid A$$
$$A \to abA \mid \varepsilon$$

   4 p.
   (b) Design an unambiguous grammar generating the same language as the grammar in part (a). *5 p.*

4. (a) Justify the claim: if languages A and B are regular, then so are the languages A^*, B^* and $A \cap B$. *3p.*
   (b) Based on the previous result, show that if $m, n \geq 1$ are arbitrary integers, then the language

$$L_{mn} = \{0^k \mid k \text{ is divisible by both } m \text{ and } n\}$$

   is regular. *4p.*

Total 40p.