Helsinki University of Technology
Laboratory for Theoretical Computer Science
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T–79.1001 Introduction to Theoretical Computer Science T (4 cr)
Exam 20 August 2007 noon to 3 p.m.

Write on every answer sheet:
- Name, degree programme, student number
- The text: "T–79.1001 Introduction to Theoretical Computer Science T, 20 August 2007"
- The total number of answer sheets submitted for grading

1. Finite state automata and regular expressions.

   a) Design a deterministic finite state automaton that recognizes the language $L = \{w \in \{a,b\}^* \mid w$ has an odd number of both as and bs$\}$ *5 p.*

   b) Design a regular expression that describes the language $L = \{w \in \{a,b\}^* \mid w$ has an odd number of as or bs (or both)$\}$ *5 p.*

   c) Find the minimal deterministic finite state automaton that accepts the language $L = \{w \in \{a,b\}^* \mid w$ has an odd number of either as or bs (but not both)$\}$ *5 p.*

2. a) Design a context-free grammar for the language

   $$L = \{ucvcw \mid u,v,w \in \{0,1\}^*, v = u^R \text{ or } v = w^R \text{ (or both)}\}.$$

   (Notation x^R denotes the reverse of string x, i.e. string x written backwards.) *5 p.*

   b) Show that the grammar you gave in part (a) is ambiguous. *5 p.*

   c) Prove (precisely!) that the language in part (a) is not regular. (*Hint:* Consider e.g. strings of the form $a^n c a^n c b^n$.) *5 p.*

3. Design a single-tape Turing machine that decides whether the input is of the form wcw, where $w \in \{a,b\}^*$. Present the computation of your machine with inputs *abab* and *abcab*. *15p.*

4. a) Closure properties of language classes. Show that if the language L is regular, then the language $\bar{L}$ (the complement of L) is also regular. *5p.*

   b) Show that if the languages L_1 and L_2 are context-free, then the language $L_1 \cup L_2$ is also context-free. *5p.*

   c) Show that if the languages L_1 and L_2 are recursively enumerable, then the language $L = L_1 \cap L_2$ is also recursively enumerable. *5p.*

Total 60p.