Helsinki University of Technology
Laboratory for Theoretical Computer Science
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T-79.1001 Introduction to Theoretical Computer Science T (4 cr)
Exam Tue 6 March 2007 9 a.m. to noon

Write on every answer sheet:
- Name, degree programme, student number
- The text: "T-79.1001 Introduction to Theoretical Computer Science T 6.3.2007"
- The total number of answer sheets submitted for grading

1. Describe the following languages **both** in terms of regular expressions **and** in terms of deterministic finite automata:

   (a) $L = \{w \in \{0,1\}^* \mid w \text{ contains } 010 \text{ as a substring}\}$; 7*p.*

   (b) $\overline{L} = \{w \in \{0,1\}^* \mid w \text{ does not contain } 010 \text{ as a substring}\}$. 8*p.*

   Hint: It may be easiest to derive the solution to (b) from the solution to (a).

2. Consider the *properly nested* strings of parentheses and angle brackets. For example, `([])[]` and `[([])]` are strings of properly nested parenthesis, but `([]`, `[)`, and `]()[` are not. More formally, the properly nested strings can be defined inductively: ε is a string of properly nested parenthesis, and if x and y are strings of properly nested parenthesis, then so are `(`x`)`, `[`y`]`, and xy.

   (a) Design a context-free grammar that produces L. 8 *p.*

   (b) Give the parse trees of the strings `([])[]` and `[([])]` in your grammar. 7 *p.*

3. Consider the language $L = \{ww \mid w \in \{a,b\}^*\}$.

   (a) Show that L is not regular. 7 *p.*

   (b) Design a Turing machine that decides L. Give your Turing machine as a state diagram, and describe the working idea of your machine in writing. 8 *p.*

4. Closure properties of language classes: Given a language L over an alphabet Σ, let $L^R = \{w^R \mid w \in L\}$ be the language obtained by reversing each string in L. Here w^R is the reverse of w (for example, $(gnat)^R = tang$).

   (a) Show that if L is regular, then L^R is regular. 5 *p.*

   (b) Show that if L is context-free, then L^R is context-free. 5 *p.*

   (c) Show that if L is recursively enumerable, then L^R is recursively enumerable. 5 *p.*

Total 60p.