1. Show that the following languages are regular by describing each of them as a regular expression or as a finite state automaton:
   (a) \( \{ w \in \{0,1\}^* \mid w \text{ starts or ends with the substring } 101 \} \), 5p.
   (b) \( \{ w \in \{a,b\}^* \mid w \text{ contains an even number of } bs \} \), 5p.
   (c) \( \{ w \in \{0,1\}^* \mid w \text{ does not contain three consecutive ones} \} \), 5p.

2. (a) Describe verbally the language produced by the following grammar:
   \[
   \begin{align*}
   S & \rightarrow ASb \mid \varepsilon \\
   A & \rightarrow aA \mid a
   \end{align*}
   \]
   5p.
   (b) Design a nonambiguous context-free grammar that produces the same language. 5p.
   (c) Show that the language produced by the above grammar is not regular. 5p.

3. Design a Turing machine that recognises the language
   \[
   L = \{ w \mid w \text{ contains equally many } as \text{ and } bs \}. 
   \]
   If you wish, your machine may have multiple tapes. Present your machine as a state diagram and describe its method of operation verbally. 15p.

4. One of the following:
   (a) Design an unrestricted grammar for the language
   \[
   L = \{ ww \mid w \in \{a,b\}^* \}. 
   \]
   (b) Show that it is an unsolvable problem to determine whether a given Turing machine \( M \), while handling the given input \( x \), writes the given character \( \sigma \) on the tape at any stage during the computation. 15p.

Total 60p.