Helsinki University of Technology
Laboratory for Theoretical Computer Science
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T-79.1001 Introduction to Theoretical Computer Science T (4 cr)
Exam Thu 21 Dec 2006 2 p.m. to 5 p.m.

Write on every answer sheet:
- Name, degree programme, student number
- The text: "T-79.1001 Introduction to Theoretical Computer Science T 21.12.2006"
- The total number of answer sheets submitted for grading

1. Show that the following languages are regular by describing each of them as a regular expression or as a finite state automaton:

   (a) $\{w \in \{0,1\}^* \mid w \text{ starts or ends with the substring } 101\}$, *5p.*

   (b) $\{w \in \{a,b\}^* \mid w \text{ contains an even number of } b\text{s}\}$, *5p.*

   (c) $\{w \in \{0,1\}^* \mid w \text{ does not contain three consecutive ones}\}$. *5p.*

2. (a) Describe verbally the language produced by the following grammar: *5p.*

$$S \longrightarrow ASb \mid \varepsilon$$
$$A \longrightarrow aA \mid a$$

   (b) Design a nonambiguous context-free grammar that produces the same language. *5p.*

   (c) Show that the language produced by the above grammar is not regular. *5p.*

3. Design a Turing machine that recognises the language

$$L = \{w \mid w \text{ contains equally many } a\text{s and } b\text{s}\}.$$

   If you wish, your machine may have multiple tapes. Present your machine as a state diagram and describe its method of operation verbally. *15p.*

4. One of the following:

   (a) Design an unrestricted grammar for the language

$$L = \{ww \mid w \in \{a,b\}^*\}.$$

   (b) Show that it is an unsolvable problem to determine whether a given Turing machine M, while handling the given input x, writes the given character σ on the tape at any stage during the computation.

15p.

Total 60p.