Helsinki University of Technology
Laboratory for Theoretical Computer Science
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T-79.1001 Introduction to Theoretical Computer Science T (4 ECTS)
Exam Thu 26 October 2006, 1–4 p.m.

Write down on each answer sheet:
- Your name, department, and student id
- The text: "T-79.1001 Introduction to Theoretical Computer Science T 26.10.2006 "
- The total number of answer sheets you are submitting for grading

1. Show that each of the following languages is regular, for example by describing them as regular expressions or finite automata.

   (a) $\{w \in \{0,1\}^* \mid |w| \geq 2, |w| \text{ is odd}\}$, *5p.*

   (b) $\{w \in \{0,1\}^* \mid |w| \geq 3, w \text{ starts with 010 or ends with 110}\}$, *5p.*

   (c) $\{w \in \{a,b,c\}^* \mid w \text{ contains neither } ab \text{ nor } cc \text{ as a substring}\}$. *5p.*

2. Consider the language $L = \{0^i 1^j 0^k \mid j = i + k\}$.

   (a) Give a context-free grammar that produces L. *7p.*

   (b) Design a pushdown automaton that recognizes L. *8p.*

3. Design a single-tape Turing machine that decides whether the input is of the form wcw, where $w \in \{a,b\}^*$. Present the computation of your machine with inputs *abab* and *abcab*. *15p.*

4. Consider strings over the alphabet $\{0,1\}$. Let $n_0(w)$ denote the number of 0s in string w. Let $L_1 = \{0^i 1^j \mid i > j \geq 0\}$ and $L_2 = \{w \mid n_0(w) \leq 3\}$. Which of the following languages are regular? Justify your answers formally.

   (a) L_1

   (b) L_2

   (c) $L_1 \cup L_2$

   (d) $L_1 \cap L_2$

   15p.

Total 60p.