1. Show that each of the following languages is regular, for example by describing them as regular expressions or finite automata.

   (a) \( \{ w \in \{0,1\}^* \mid |w| \geq 2, |w| \text{ is odd} \} \), \hspace{1cm} 5p.

   (b) \( \{ w \in \{0,1\}^* \mid |w| \geq 3, w \text{ starts with 010 or ends with 110} \} \), \hspace{1cm} 5p.

   (c) \( \{ w \in \{a,b,c\}^* \mid w \text{ contains neither } ab \text{ nor } cc \text{ as a substring} \} \). \hspace{1cm} 5p.

2. Consider the language \( L = \{0^i1^j0^k \mid j = i+k \} \).

   (a) Give a context-free grammar that produces \( L \). \hspace{1cm} 7p.

   (b) Design a pushdown automaton that recognizes \( L \). \hspace{1cm} 8p.

3. Design a single-tape Turing machine that decides whether the input is of the form \( wcw \), where \( w \in \{a,b\}^* \). Present the computation of your machine with inputs \( abab \) and \( abcab \). \hspace{1cm} 15p.

4. Consider strings over the alphabet \( \{0,1\} \). Let \( n_0(w) \) denote the number of 0s in string \( w \). Let \( L_1 = \{0^i1^j \mid i > j \geq 0 \} \) and \( L_2 = \{ w \mid n_0(w) \leq 3 \} \). Which of the following languages are regular? Justify your answers formally.

   (a) \( L_1 \) \hspace{1cm} 15p.

   (b) \( L_2 \)

   (c) \( L_1 \cup L_2 \)

   (d) \( L_1 \cap L_2 \)