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Laboratory for Theoretical Computer Science
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T-79.1001 Introduction to Theoretical Computer Science T (4 ECTS)
Exam Wed 30 Aug 2006, 1–4 p.m.

Write down on each answer sheet:
- Your name, department, and student id
- The text: "T-79.1001 Introduction to Theoretical Computer Science T 30.8.2006"
- The total number of answer sheets you are submitting for grading

This exam corresponds to the pre-2005 course T-79.148.

1. Show that each of the following languages is regular, by describing it either in terms of a regular expression or in terms of a finite automaton:

   (a) $\{w \in \{0,1\}^* \mid |w| \geq 2,\ w \text{ begins and ends with the same symbol}\}$, *5p.*

   (b) $\{w \in \{0,1\}^* \mid |w| \geq 3,\ w \text{ ends in either string 010 or string 110}\}$, *5p.*

   (c) $\{w \in \{0,1\}^* \mid w \text{ does not contain substring 1101}\}$. *5p.*

2. (a) Design a context-free grammar for the language

   $$L = \{ucvcw \mid u,v,w \in \{0,1\}^*,\ v = u^R \text{ or } v = w^R \text{ (or both)}\}.$$

   (Notation x^R denotes the reverse of string x, i.e. string x written backwards.) *5 p.*

   (b) Show that the grammar you gave in part (a) is ambiguous. *5 p.*

   (c) Prove (precisely!) that the language in part (a) is not regular. (*Hint:* Consider e.g. strings of the form $0^n c 0^n c 1^n$.) *5 p.*

3. Design a deterministic single-tape Turing machine that checks that the binary string it receives as input contains more ones than zeros. (Present the machine preferably in the form of a state diagram rather than as a transition table.) Show the accepting computation sequence ("run") of your machine on input 011, and the rejecting computation sequence on input 1010. *15p.*

4. *One* of the following:

   (a) Prove that if the languages $L \subseteq \{0,1,\#\}^*$ and $L' \subseteq \{0,1\}^*$ are context-free, then so is the language $L'' = L[L'] \subseteq \{0,1\}^*$, whose words are obtained from the words in L by replacing each #-symbol by some word in L' (not necessarily always the same). *15p.*

   (b) Assume that you are explaining the key contents of the course "Introduction to Theoretical Computer Science T" to a friend who has not yet taken the course. Describe the Church-Turing thesis to her, and convince her of the fact that there are problems that cannot be solved by a computer. *15p.*

Total 60p.