

Helsinki University of Technology
Laboratory for Theoretical Computer Science

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T-79.1001 Introduction to Theoretical Computer Science T (4 ECTS)

Exam Wed 14 Dec 2005, 1–4 p.m.

Write down on each answer sheet:

- Your name, department, and student id
 - The text: “T-79.1001 Introduction to Theoretical Computer Science T 14.12.2005”
 - The total number of answer sheets you are submitting for grading
- This exam corresponds to the pre-2005 course T-79.148.

1. (a) Give a regular expression that describes the language

$$\{w \in \{0, 1\}^* \mid w \text{ contains an odd number of 0's or an odd number of 1's (or both).}\}$$

5p.

- (b) Design a nondeterministic finite automaton with no ϵ -transitions that recognises the language in part (a). 5p.

- (c) Design a deterministic finite automaton that recognises the language in part (a). 5p.

2. (a) Design a context-free grammar for the language

$$L = \{a^i b^j c^k \mid i = j \text{ or } j = k\}.$$

5 p.

- (b) Show that the grammar you gave in part (a) is ambiguous. 5 p.

- (c) Prove (precisely!) that the language in part (a) is not regular. 5 p.

3. Design a (nondeterministic) pushdown automaton that recognises (accepts) the language L considered in problem 2. (Present the automaton preferably as a state diagram rather than a transition table.) Show the accepting computation sequences (“runs”) of your automaton machine on inputs ab and $abbcc$. 15p.

4. *One* of the following:

- (a) Justify the claim: if language $L \subseteq \{0, 1\}^*$ is context-free, then so is the following language that consists of all the prefixes of the words in L :

$$L_{\text{pref}} = \{x \in \{0, 1\}^* \mid xy \in L \text{ for some } y \in \{0, 1\}^*\}.$$

(*Hint:* It may be useful to consider grammars in Chomsky normal form here.) 15p.

- (b) Explain what is meant by an undecidable problem, give an example of such, and prove that your example problem indeed is undecidable. You may use any results in your study material except ones that directly state that “problem P is undecidable”. Give precise definitions and arguments. 15p.

Total 40p.