T–79.1001/2 **Autumn 2007**
Introduction to Theoretical Computer Science T/Y
Tutorial 6, 18 to 23 October
Problems

This is the last lecture and exercise round of the short course T–79.1002; the exam on Oct 25 covers topics covered in lectures and exercises on weeks 1–6. Those participating in the longer course T–79.1001 should take the December exam, not the October exam.

Participants of T–79.1002 are kindly asked to give course feedback via the WWW page of the course after the exam by Nov 5.

The course T–79.1001 continues with the next lectures (still in Finnish) on Oct 23 and Nov 6 and with tutorials in English on Nov 1 and Nov 8.

Homework problems:

1. Consider the following grammar generating a certain type of list structures:

$$S \to (S) \mid S, S \mid a.$$

   (a) Based on the above grammar, give a leftmost and rightmost derivation and a parse tree for the sentence "$(a, (a))$".

   (b) Prove that the grammar is ambiguous.

   (c) Design an unambiguous grammar generating the same language.

2. Construct context-free grammars for the following languages:

   (a) $\{ucv \mid u, v \in \{a, b\}^* \text{ and } |u| = |v|\}$,

   (b) $\{a^m b^n \mid m > n\}$,

   (c) $\{a^m b^n \mid m \neq n\}$.

   Additionally, give a derivation for the string *abcab* using your first grammar and another for *aaabb* using your second grammar. For the third grammar, note that $m \neq n$ if and only if $m > n$ or $m < n$.

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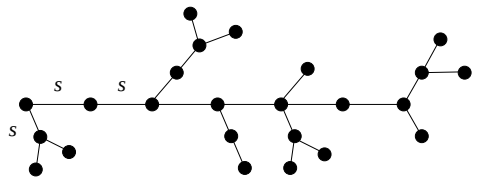
3. A *fern* consists of a stem and a number of subferns rooted on the left and right sides of the stem. For instance, the following structure is a fern:



A fern structure can be described by a string where each unit of the stem is denoted by a letter s, and each subfern is described by a similar string in parentheses, located at the point where the subfern is rooted, and prefixed by l or r depending on whether the subfern occurs on the left or right side of the main stem, respectively. At most one subfern can be rooted to the left and to the right at each point, and each subfern must contain at least one stem unit. For instance, the string representation corresponding to the above example would be:

$$r(sl(s)r(s))ssl(ssl(s)r(s))sr(ss)sl(s)r(sl(s)r(s))ssl(sr(s)s)r(s).$$

Design a context-free grammar describing the structure of such fern strings.

Demonstration problems:

4. Design a context-free grammar for the language

$$\{w \in \{a, b\}^* \mid w \text{ contains equally many } a\text{'s and } b\text{'s}\} \ .$$

5. (a) Prove that the following context-free grammar is ambiguous:

$$\begin{aligned} S &\rightarrow \textbf{if } b \textbf{ then } S \\ S &\rightarrow \textbf{if } b \textbf{ then } S \textbf{ else } S \\ S &\rightarrow s. \end{aligned}$$

   (b) Design an unambiguous grammar that is equivalent to the grammar in item (a), i.e. that generates the same language. (*Hint:* Introduce new nonterminals B and U that generate, respectively, only "balanced" and "unbalanced" **if-then-else**-sequences.)

6. Design a recursive-descent (top-down) parser for the grammar from Problem 5 of the previous tutorial.