Homework problems:

1. Construct a nondeterministic finite automaton that tests whether a given binary input sequence contains 101 or 011 (or both) as a subsequence. Make the automaton deterministic using the subset construction.

2. Construct the minimal automaton corresponding to the following deterministic finite automaton:

```
1 ----> 2
|      | a
|      v
b      b
      3 ----> 4
```

3. Show that if a language \( L \subseteq \{a, b, c\}^* \) is recognised by some finite automaton, then so is the language \( L |\{a, b\} \), which is obtained by removing all \( c \)'s from each string in \( L \).

Demonstration problems:

4. Construct a nondeterministic finite automaton that tests whether in a given binary input sequence the third-to-last bit is a 1. Make the automaton deterministic using the subset construction.

5. Show that if a language \( L \subseteq \{a, b\}^* \) is recognised by some finite automaton, then so is the language \( L^R = \{w^R \mid w \in L\} \). (The notation \( w^R \) means the reverse of string \( w \), that is, the string where the characters of \( w \) are in reverse order.)

6. Show that if languages \( A \) and \( B \) over the alphabet \( \Sigma = \{a, b\} \) are recognised by some finite automata, then so are the languages \( \bar{A} = \Sigma^* - A \), \( A \cup B \), and \( A \cap B \).