T–79.1001/2 **Autumn 2007**

Introduction to Theoretical Computer Science T/Y

Tutorial 1, 13 to 18 September

Problems

Remember to enroll for the course using the TOPI registration system by 26 September. For bookkeeping reasons, registration is **compulsory***, even if you were not intending to attend the lectures or the tutorial sessions.*

Homework problems:

1. Let $A = \{a, b, e\}$, $B = \{b, d\}$, and $C = \{a, c, d\}$. List the elements of the following sets:
   (a) $A \cup (C - B)$;
   (b) $B \times (A \cap C)$;
   (c) $\mathcal{P}(\{\emptyset\}) \times \mathcal{P}(\emptyset)$.

2. Let $\Sigma = \{a, b\}$. Give some examples of strings from each of the following languages (at least three strings per language):
   (a) $\{w \in \Sigma^* \mid w$ contains at least two a's and the number of b's is divisible by three$\}$;
   (b) $\{a^{2n}b^{3m} \mid 0 \leq n < m\}$;
   (c) $\{uvvu \mid u, v \in \Sigma^*\}$;
   (d) $\{w \in \Sigma^* \mid \exists u, v \in \Sigma^*$ s.t. $w = uu = vvv\}$.

3. Verify by induction the correctness of the formula:
$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

Demonstration problems:

4. Define a relation $\sim$ on the set $\mathbb{N} \times \mathbb{N}$ by the rule:
$$(m, n) \sim (p, q) \quad \Leftrightarrow \quad m + n = p + q.$$
   Prove that this is an equivalence relation, and describe intuitively ("geometrically") the equivalence classes it determines.

5. Prove by induction that if X is a finite set of cardinality $n = |X|$, then its power set $\mathcal{P}(X)$ is of cardinality $|\mathcal{P}(X)| = 2^n$.

6. Prove by induction that every partial order defined on a finite set S contains at least one minimal element. Furthermore, provide examples showing that the minimal element is not necessarily unique (i.e. there can be more than one), and that in an infinite set S the claim does not necessarily hold.